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AUTHOR DeVenney, William S.; And Others
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ABSTRACT

This text is one of the sequence of textbooks produced for low achievers in the seventh and eighth grades by the School Mathematics Study Group (SMSG). There are eight texts in the sequence, of which this is the fifth. This set of volumes differs from the regular editions of SMSG junior high school texts in that very little reading is required. Concepts and processes are illustrated pictorially, and many exercises are included. This volume deals with decimals (chapter 10) and parallelism (chapter 11). After a brief review of the fundamental operations on whole numbers, the place value system and use of decimal notation are discussed. The decimal point is introduced in the context of the monetary system, and exercises involving conversion from decimal to common fractions, and conversely, are presented. The chapter on parallelism begins with a review of congruence, and relies on constructions in developing the notions of perpendicularity and parallelism. This volume includes tables for addition and multiplication, and flow charts for operations on rationals to be used by students as needed. (SD)

SECONDARY SCHOOL MATHEMATICS

SPECIAL EDITION

Chapter 10. Decimals

Chapter 11. Parallelism

MEMBERS OF WRITING TEAMS

William S. DeVenney, Ashland, Oregon
James C. McCaig, Cupertino School District, Cupertino, Calif.
Jane G. Stenzel, Cambrian Elementary School District, San Jose, Calif.

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Addition Table (continued)

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Multiplication Table

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5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225
16	0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240
17	0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255
18	0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270
19	0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285
20	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300
21	0	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315
22	0	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330
23	0	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345
24	0	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360
25	0	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375
26	0	26	52	78	104	130	156	182	208	234	260	286	312	338	364	390
27	0	27	54	81	108	135	162	189	216	243	270	297	324	351	378	405
28	0	28	56	84	112	140	168	196	224	252	280	308	336	364	392	420
29	0	29	58	87	116	145	174	203	232	261	290	319	348	377	406	435
30	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450

Multiplication Table (continued)

X	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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2	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60
3	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90
4	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120
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6	96	102	108	114	120	126	132	138	144	150	156	162	168	174	180
7	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210
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9	144	153	162	171	180	189	198	207	216	225	234	243	252	261	270
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11	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330
12	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360
13	208	221	234	247	260	273	286	299	312	325	338	351	364	377	390
14	224	238	252	266	280	294	308	322	336	350	364	378	392	406	420
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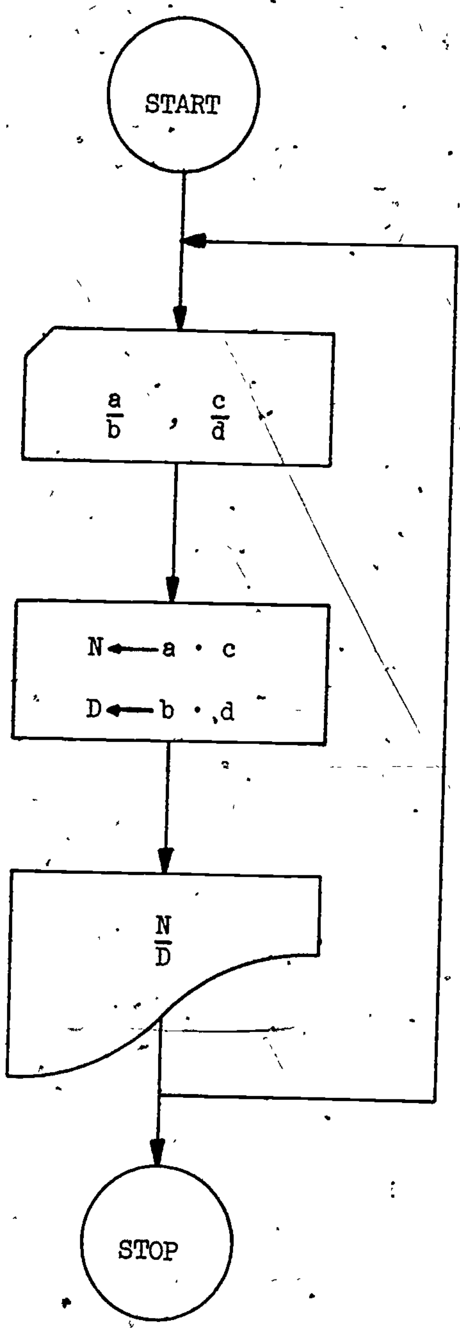
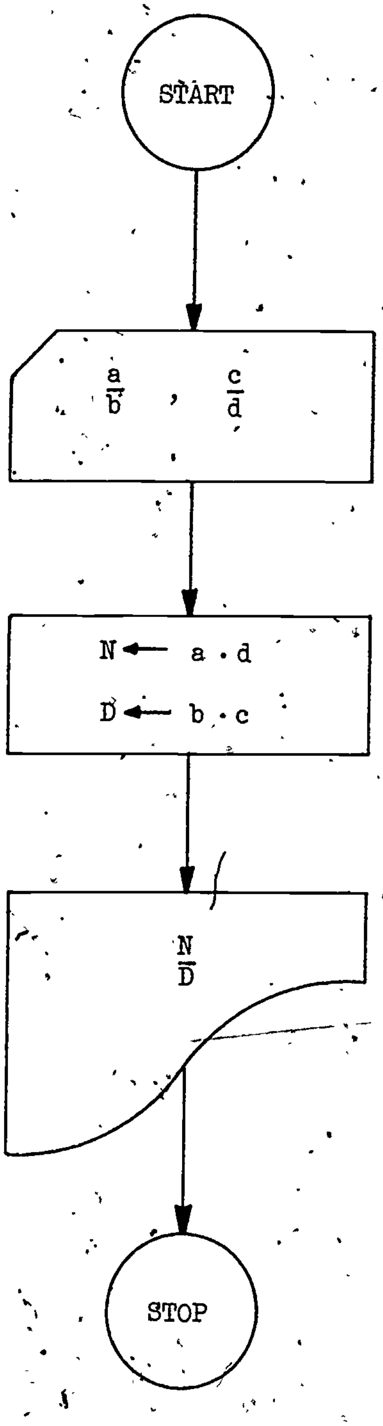
17	272	289	306	323	340	357	374	391	408	425	442	459	476	493	510
18	288	306	324	342	360	378	396	414	432	450	468	486	504	522	540
19	304	323	342	361	380	399	418	437	456	475	494	513	532	551	570
20	320	340	360	380	400	420	440	460	480	500	520	540	560	580	600
21	336	357	378	399	420	441	462	483	504	525	546	567	588	609	630
22	352	374	396	418	440	462	484	506	528	550	572	594	616	638	660
23	368	391	414	437	460	483	506	529	552	575	598	621	644	667	690
24	384	408	432	456	480	504	528	552	576	600	624	648	672	696	720
25	400	425	450	475	500	525	550	575	600	625	650	675	700	725	750
26	416	442	468	494	520	546	572	598	624	650	676	702	728	754	780
27	432	459	486	513	540	567	594	621	648	675	702	729	756	783	810
28	448	476	504	532	560	588	616	644	672	700	728	756	784	812	840
29	464	493	522	551	580	609	638	667	696	725	754	783	812	841	870
30	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900

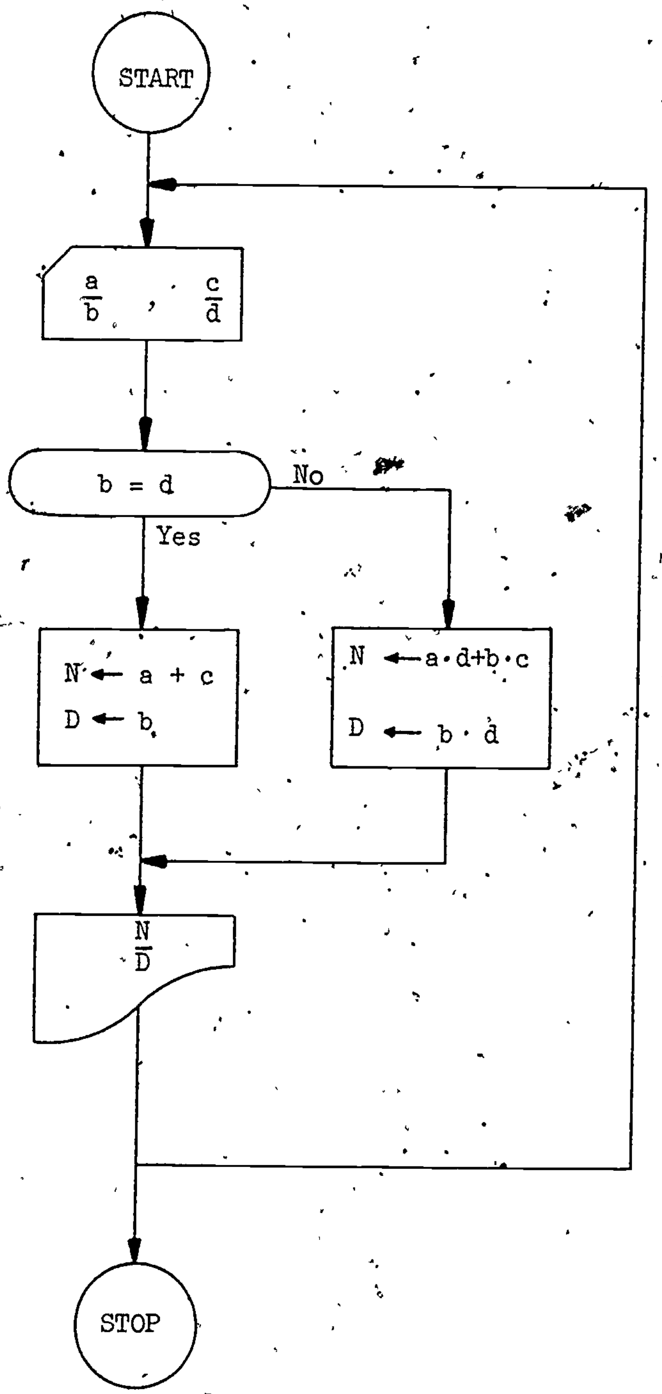
A Flow Chart for Multiplication of Fractions



A Flow Chart for Division of Fractions
($\frac{c}{d}$ the divisor)

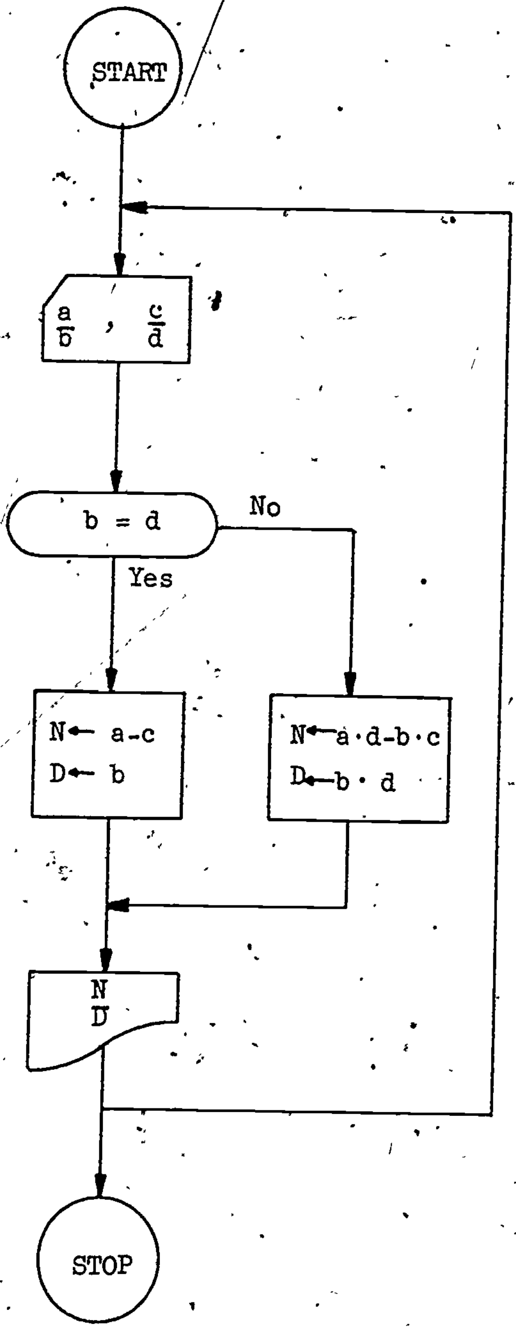


A Flow Chart for Addition of Fractions



A Flow Chart for Subtraction of Fractions

$(\frac{a}{b} > \frac{c}{d}$



Chapter 10

DECIMALS

DECIMALS

Introduction

In the lower grades you did most of your work with counting numbers and whole numbers. In earlier chapters of this book you learned how to work with integers and rational numbers. You have already seen how place value and the use of powers of ten can help you in multiplying and dividing whole numbers. Now we will learn how place value and powers of ten help in working with rational numbers.

To begin, we see how to use what you have already learned in order to divide integers quickly and easily.

Division With Integers

You know that you can think of multiplication this way:

7×3 means 7 rows with 3 things in each row.

```
* * *
* * *
* * *
* * *
* * *
* * *
* * *
```

You also think of multiplication as repeated addition:

$$3 \times 7 = 7 + 7 + 7$$

or

$$= 3 + 3 + 3 + 3 + 3 + 3 + 3 .$$

Either way, you find that $7 \times 3 = 21$.

You can think of division in different ways, too. "21 divided by 7 = 3" ($\frac{21}{7} = 3$) means, "If you put 21 things into 7 rows, you will have 3 things in each row."

You can think of $\frac{21}{7}$ as asking $7 \times \underline{\quad ? \quad} = 21$.

You can think of division as repeated subtraction. "How many times can you subtract 7 from 21 ?"

$$\begin{array}{r} 21 \\ \underline{-7} \\ 14 \\ \underline{-7} \\ 7 \\ \underline{-7} \\ 0 \end{array}$$

Sometimes you think of dividing just one thing into parts, as we did in Chapter 6, Rational Numbers. When you divide numbers, the answer is not always an integer.

In Chapter 5, Number Theory, you learned some of the rules of divisibility. There are numbers that are "not divisible" by 2 . This means that when you divide by 2 , you have a remainder, and if you want equal parts, you must divide the remainder by 2 .

$$\frac{9}{2} = 4\frac{1}{2}.$$

As we talk about division in this chapter, we will use a combination of these ideas.

How can we find $\frac{22}{7} = ?$ There is no whole number n that makes this sentence true:

$$7 \times n = 22 .$$

If we try to put 22 things into 7 rows, we don't have the same number of things in each row. We would have 1 left over after making 7 rows of 3 .

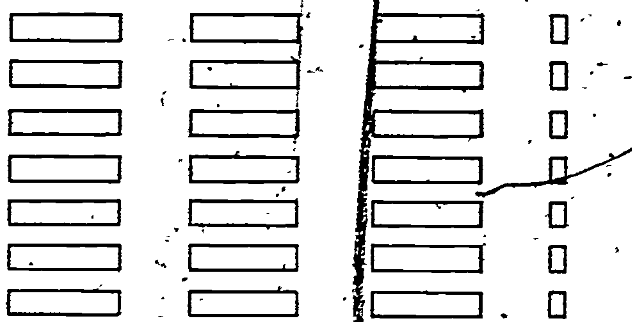
If we continue subtracting as long as we get a number greater than the divisor,

$$\begin{array}{r} 22 \\ \underline{-7} \\ 15 \\ \underline{-7} \\ 8 \\ \underline{-7} \\ 1 \end{array}$$

we have 1 left over. With remainders, we think of dividing the "left-overs" into parts that are less than 1 . In the problem $\frac{22}{7}$ we divide the left-over 1 into 7 equal parts and say that

$$\frac{22}{7} = 3\frac{1}{7}.$$

If we think of putting 22 things into 7 rows, we could do the same kind of thing.



Each little rectangle is $\frac{1}{7}$ of the left-over big one.

Although subtracting again and again is one way to solve a division problem, it would take a long time to work a problem like $\frac{700}{7}$ by subtracting 7 over and over until you had a remainder that was less than 7. Luckily, you know that 100×7 is 700. You can subtract 100 sevens all at once!

$$\frac{700}{7} = 100$$

If you have $\frac{70}{7}$, you can subtract 10 sevens because you know that $10 \times 7 = 70$.

$$\frac{70}{7} = 10$$

If you have $\frac{770}{7}$, you first think of subtracting 100 sevens, which would leave 70, and then of subtracting 10 more sevens, so

$$\frac{770}{7} = 110 .$$

What can you do with $\frac{210}{7}$? You know that $3 \times 7 = 21$, so $30 \times 7 = 210$. You can subtract 30 sevens.

$$\frac{210}{7} = 30$$

Class Discussion

1. Rewrite this division problem $\frac{486}{2} = ?$ like this: $2\overline{)486}$. (Either way you write it, the dividend is 486; the divisor is 2.

   (a) What does the 4 in 486 mean? ____________________

   (b) $2 \times$ ____________ $= 4$

   (c) $2 \times$ ____________ $= 400$

   To show that you can subtract 200 twos from 486, write 2 in the hundreds place of your answer:

$$\begin{array}{r} 2 \\ 2\overline{)486} \end{array}$$

   (d) When you subtract 200 twos from 486, what is left?

   ____

   (e) $2 \times$ ____________ $= 8$

   (f) $2 \times$ ____________ $= 80$

   Write 4 in the tens place in your answer.

   (g) When you subtract 80 from 86, what is left? ____________

   (h) $2 \times$ ____________ $= 6$

   Write 3 in the ones place in your answer.

   (i) When you subtract 6 from 6, what is left? ____________

   (j) $\frac{486}{2} =$ ____________________

2. Rewrite this division problem $\frac{384}{2} = ?$ like this: $2\overline{)384}$.

   (a) What does the 3 in the dividend mean? ____________

   (b) Can you subtract 200 twos from 300? ____________

   (c) Can you subtract 100 twos from 300? ____________

   (d) $100 \times 2 =$ ____________

   Write 1 in the hundreds place in your answer.

(e) When you subtract 200 from 300, what is left? ____________

Write 1 up in front of the 8 in the dividend as a reminder that you have 100 left over.

$$2\overline{)3^{1}84}$$

(f) What is left when you subtract 100 twos from 384 ? ____________

(g) 100 + 80 = ____________

(h) 2 × ____________ = 18

(i) 2 × ____________ = 180

Write 9 in the tens place in your answer.

(j) What is left after you subtract 180 ? ____________

(k) 2 × ____________ = 4

Write 2 in the ones place in your answer.

(l) $\frac{384}{2}$ = ____________

3. Rewrite this division problem $\frac{3633}{7} = ?$ like this: $7\overline{)3633}$.

(a) Can you subtract 1000 sevens from 3000 ? ____________

(b) Can you find 36 in the 7 row of your multiplication table? ____________

(c) What is the largest product that is less than 36 in that row? ____________

(d) 7 × ____________ = 35 so 7 × ____________ = 3500.

To show that you can subtract 500 sevens, write 5 in the hundreds place in your answer.

(e) When you subtract 3500 from 3600, how many hundreds are left? ____________

Write 1 up in front of 33 in the dividend to remind you that you have 133 left to subtract from.

(f) Can you find 13 in the seven row of your multiplication table? ____________

(g) What is the largest product that is less than 13 in that row? ____________

(h) $7 \times$ ________ $= 7$, so $7 \times$ ________ $= 70$.

Write 1 in the tens place in your answer.

(i) $13 - 7 =$ ________, so $130 - 70 =$ ________.

Write 6 in front of the last 3 in the dividend to remind you that you have 63 left to subtract from. Your problem should now look like this:

$$\begin{array}{r} 5\ 1\phantom{\ {}^{6}3} \\ 7\,\overline{)\,3\ 6\ {}^{1}3\ {}^{6}3} \end{array}$$

(j) $7 \times$ ________ $= 63$

Write 9 in the ones place in your answer.

(k) $\frac{3633}{7} =$ ____________

4. The steps in dividing $\frac{5439}{4} = ?$ are shown below. Make sure you understand each step. Notice that the remainder is written as a fraction.

(a) $$\begin{array}{r} 1\phantom{\,{}^{1}4\,3\,9} \\ 4\,\overline{)\,5\,{}^{1}4\,3\,9} \end{array}$$ 1000 fours and 1000 left over.

(b) $$\begin{array}{r} 1\;3\phantom{\,{}^{2}3\,9} \\ 4\,\overline{)\,5\,{}^{1}4\,{}^{2}3\,9} \end{array}$$ 300 fours and 200 left over.

(c) $$\begin{array}{r} 1\;3\;5\phantom{\,{}^{3}9} \\ 4\,\overline{)\,5\,{}^{1}4\,{}^{2}3\,{}^{3}9} \end{array}$$ 50 fours and 30 left over.

(d) $$\begin{array}{r} 1\;3\;5\;9\tfrac{3}{4} \\ 4\,\overline{)\,5\,{}^{1}4\,{}^{2}3\,{}^{3}9} \end{array}$$ 9 fours and 3 left over.

Exercises

Write any remainder as a fraction.

1. $3\overline{)363}$
2. $4\overline{)848}$
3. $3\overline{)499}$
4. $4\overline{)648}$
5. $6\overline{)4882}$
6. $8\overline{)6896}$
7. $6\overline{)4928}$
8. $9\overline{)6524}$
9. $8\overline{)7932}$
10. $11\overline{)8492}$
11. $9\overline{)5015}$
12. $7\overline{)6111}$

Division by Numbers Greater Than 10

The division problems you did in the last lesson could be done without writing down all the subtraction and multiplication answers. You could do the work mentally. When you have divisors that are greater than 10, however, it is harder to do the work without writing things down. In the exercises that follow, notice that you follow the same thinking you did before, but, to make the work easier, you write more.

Class Discussion

1. Rewrite $\frac{1675}{25}$ like this: $25\overline{)1675}$.

   (a) What does the 1 in the dividend mean? ______

   (b) Can you subtract 1000 twenty-fives from 1000? ______

   (c) Can you subtract 100 twenty-fives from 1000? ______

   (d) Look in the 25 row of your multiplication table. Can you find 167 in the products in that row? ______

   (e) What is the largest product that is less than 167 in that row? ______

   (f) $25 \times$ ______ $= 150$, so $25 \times$ ______ $= 1500$.

   Write 6 in the tens place in your answer. To find out how much is left to subtract from, write 1500 under 1675 and subtract. Your problem should look like this:

$$\begin{array}{r} 6 \\ 25\overline{)1675} \\ -\,1500 \\ \hline 175 \end{array}$$

   If you had subtracted mentally, it would look like this:

$$\begin{array}{r} 6 \\ 25\overline{)\cancel{1}\,6^{1}7\,5} \end{array}$$

(g) $25 \times$ __________ $= 175$

Write 7 in the ones place in your answer.

(h) $\frac{1675}{25} =$ _______________

2. The steps in dividing $\frac{14991}{21} = ?$ are shown below. Explain each step.

(a)
$$\begin{array}{r} 7 \\ 21\overline{)14991} \\ -14700 \\ \hline 291 \end{array}$$

$21 \times$ __________ $= 14700$

The 7 in the answer means _______________.

(b)
$$\begin{array}{r} 71 \\ 21\overline{)14991} \\ -14700 \\ \hline 291 \\ -210 \\ \hline 81 \end{array}$$

$21 \times$ __________ $= 210$

The 1 in the answer means _______________.

(c)
$$\begin{array}{r} 713 \\ 21\overline{)14991} \\ -14700 \\ \hline 291 \\ -210 \\ \hline 81 \\ -63 \\ \hline 18 \end{array}$$

$21 \times$ __________ $= 63$

The 3 in the answer means _______________.

(d)
$$\begin{array}{r} 713\frac{18}{21} \\ 21\overline{)14991} \\ -14700 \\ \hline 291 \\ -210 \\ \hline 81 \\ -63 \\ \hline 18 \end{array}$$

Is $\frac{18}{21}$ in simplest form? _______________

$\frac{18}{21} =$ ____________________

(e) $\frac{14991}{21} =$ ____________________

Generally, if the divisor is less than 10 , it is easier to do the work mentally and write a reminder of the "left-overs" in the dividend. But if the divisor is greater than 10 , it is easier to do the subtraction on paper. In that case, "left-overs" appear underneath the dividend.

Exercises

Use whichever method you choose for these division problems.

1. $9\overline{)1233}$

2. $8\overline{)9683}$

3. $4\overline{)26547}$

4. $7\overline{)7983}$

5. $30\overline{)1628}$

6. $44\overline{)914}$

The Decimal Point

You have used the decimal point to show a whole number and a number that is less than one whenever you have worked with dollars and cents. The number $4.65 means four dollars and sixty-five one-hundredths of a dollar, and the decimal point separates the number of whole dollars from the amount of money that is less than one dollar.

We often use fraction names when we talk about money, but we usually use decimal names when we write amounts of money. We say "a quarter" or "a half dollar", and we understand that a dime is one tenth of a dollar, but we write $.25 or $.50 or $.10, especially if we are writing a number of dollars too. We say "a dollar and a half", but we write $1.50. This makes adding and subtracting amounts of money very simple.

Exercises

Use the table below to change the amounts of money shown in fractions to the usual dollars-and-cents form. Then solve the problem by adding or subtracting. The first one is done for you.

Fraction of a dollar	Amount of money
$\frac{1}{20}$	1 nickel, or $.05
$\frac{1}{10}$	1 dime, or $.10
$\frac{1}{5}$	20 cents, or $.20
$\frac{1}{4}$	1 quarter, or $.25
$\frac{1}{2}$	1 half dollar, or $.50
$\frac{1}{100}$	1 penny, or $.01

How Much Money Is

1. $\frac{3}{4}$ dollar and $\frac{9}{20}$ dollar and $\frac{3}{100}$ dollar?

$$(3 \times \tfrac{1}{4}) \quad + \quad (9 \times \tfrac{1}{20}) \quad + \quad (3 \times \tfrac{1}{100}) =$$

$$(3 \times .25) \quad + \quad (9 \times .05) \quad + \quad (3 \times .01) =$$

$.75 and $.45 and $.03 = $1.23

2. How much is $\frac{19}{20}$ dollar and $\frac{3}{5}$ dollar? __________

3. How much is $\frac{3}{10}$ dollar and $\frac{4}{5}$ dollar and $\frac{1}{2}$ dollar? __________

4. How much is $\frac{7}{20}$ dollar and .49 dollars? __________

5. How much is $\frac{3}{4}$ dollar and $\frac{1}{5}$ dollar? __________

6. How much is 8 dollars, $\frac{4}{5}$ dollar and $\frac{1}{10}$ dollar? __________

7. How much is three dollars and a half, and ten dollars and a quarter? ______________________

8. Had $\frac{4}{5}$ dollar. Spent $\frac{1}{2}$ dollar. How much is left? ________

9. Had $\frac{9}{10}$ dollar. Spent $\frac{1}{4}$ dollar. How much is left? ________

10. Had $\frac{3}{20}$ dollar. Spent $\frac{1}{10}$ dollar. How much is left? ________

The "Times 10" Machine

In Chapter 4 you learned that when you multiply a whole number by 10 you just "tack on" a zero. When you multiply by 100, you tack on two zeros, and so on.

This works because of our place value system. When we work with whole numbers, the digit farthest to the right shows the number of ones; the next digit to the left shows the number of tens; the next digit to the left shows the number of hundreds, and so on. When a number is multiplied by ten it "moves over" one place to the left.

Class Discussion

63 means _____ tens and _____ ones.

630 means _____ hundreds, _____ tens, and _____ ones.

The digit 6 in 630 has _____ times the value of the 6 in 63.

The digit 3 in 630 has _____ times the value of the 3 in 63.

In money, \$63.00 has _____ times the value of \$6.30. The number of whole dollars in \$63.00 is _____, because that is the number to the left of the _______________.

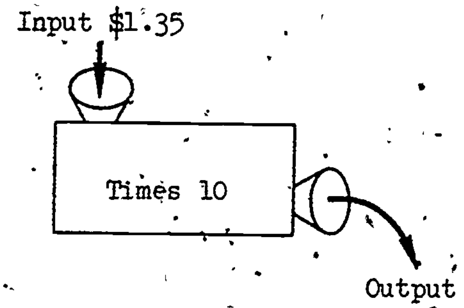
With \$6.30, however, we have only _____ whole dollars and $\frac{3}{10}$ of another dollar. If we write \$.63, the _____ means $\frac{6}{10}$ of a dollar and the _____ means $\frac{3}{100}$ of a dollar.

If we arrange this information neatly, you can see what happens.

				\$ .63
10	times	\$ .63	is	\$ 6.30
10	times	\$ 6.30	is	\$ 63.00
10	times	\$63.00	is	\$630.00

Each time when we multiplied by 10, the decimal point moved one place farther to the right, and each time it separated the _________ number of dollars from the part of a dollar less than 1.

Suppose you put $1.35 into a "times ten" function machine.



Write the amount of the input and output. ________ ________
Input Output

What happened to the decimal point? ______________________________

____

Finish the list of inputs and outputs for this machine.

$f : x \longrightarrow 10x$	
Input (dollars)	Output (dollars)
1.35	13.50
2.18	______
.57	______
4.75	______
3.14	______

With whole numbers, when you multiply by 100 you "tack on" two zeros. With money, when you multiply by 100 (which is 10 × 10) the decimal point moves _________ places to the right.

Exercises

Suppose you put each of these amounts of money into a "Times 100" machine. Finish the list of inputs and outputs.

$f : x \longrightarrow 100x$	
Input (dollars)	Output (dollars)
1.35	135.00
2.18	_________
.57	_________
4.75	_________
10.00	_________
3.14	_________
.49	_________
16.83	_________
.02	_________

Place Value and the Number 10

You have seen how easy it is to multiply amounts of money by 10 simply by moving the decimal point one place farther to the right. This can be done even when the amount of money is less than one dollar; for example, $.03 times 10 is $.30 . (We could write $0.30 , but we usually leave it $.30 .)

Since $9.85 times 10 is $98.50 we know that $98.50 divided by 10 is $9.85 . Since $.03 times 10 is $.30 , then $.30 divided by 10 is $.03 . You see that to divide by 10 you move the decimal point one place to the left, writing in a zero if necessary.

The same method is used with numbers that have nothing to do with money. We usually write whole numbers like this:

59
642
1249
7

We can, however, write:

59.
642.
1249.
7.

<u>Any whole number can be written with a decimal point to the right of the ones place.</u>

Class Discussion

Here is a list of some powers of 10 .

10.

100.

1000.

10000.

To get from any of these numbers to the one underneath it, we move the _________ _________ one place to the _________.

This is multiplying by _________.

Suppose we start at 1000 and go to 100. What happens to the decimal point? ____

This is dividing 1000 by _________.

Divide 100 by 10 . _________

To divide 100 by 10 we move the decimal point _________ place to the _________.

If we divide 10 by 10 we get _________.

We can get 1 by moving the decimal point like this: 1.0

Divide 1 by 10 . Do you get $\frac{1}{10}$? _________

When we move the decimal point one place to the left, we get .1 . So $\frac{1}{10}$ and .1 are different ways of writing the same number.

Here is a longer list of powers of 10 . Each number can be written two different ways, one with a decimal point and one without.

$$.001 = \frac{1}{1000}$$

$$.01 = \frac{1}{100}$$

$$.1 = \frac{1}{10}$$

$$1. = 1$$

$$10. = 10$$

$$100. = 100$$

$$1000. = 1000$$

$$10000. = 10000$$

The first place on the right of the decimal point is called <u>tenths</u> place, because .1 and _____ are the same number.

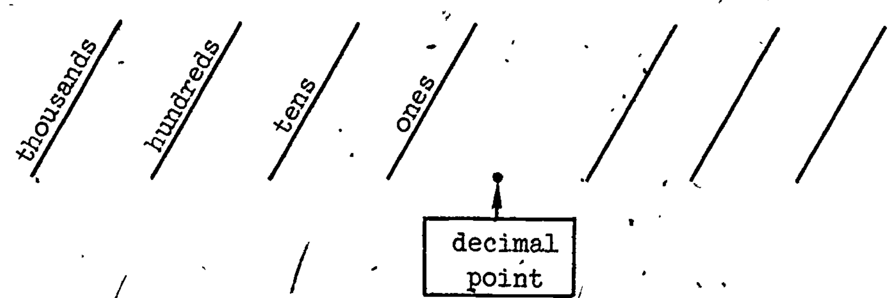
$.01 = \frac{1}{100}$ so the name of the second place to the right of the decimal point is ______________ place.

The name of the third place to the right of the decimal point is ________________________ place, because .001 and ____________ are the same number.

Notice that the place value names for numbers less than one all end in <u>th</u>.

Exercises

1. Finish filling in the names of the places shown by these blanks.



2. Put the decimal point in the following numbers so that:

(a)	the 5 is in the ones place.	3 4 5 1 3
(b)	the 4 is in the tenths place	7 3 0 4
(c)	the 6 is in the tens place	8 4 7 6 0 9
(d)	the 7 is in the hundreds place	3 7 1 1 6 6 1
(e)	the 0 is in the ones place	6 0 4 3 4
(f)	the 2 is in the hundredths place	7 5 8 6 0 2
(g)	the 9 is in the thousandths place	2 8 7 6 5 9

(h)	the 4 is in the hundreds place	1 3 4 9 5
(i)	the 0 is in the hundredths place	2 3 7 9 0 1
(j)	the 7 is in the tenths place	4 5 2 6 7 3
(k)	the 4 is in the tens place	8 4 7 5 1
(ℓ)	the 8 is in the ones place	1 0 0 8 9 5 1 6
(m)	the 2 is in the hundredths place	8 9 8 4 2 3
(n)	the 6 is in the ones place	6 8 3 9 0 5 2

Comparing Decimal Numbers

When numbers are written with a decimal point, that small dot is a very important mark. You have noticed this with money, of course. Certainly you would rather have 8 whole dollars ($8.00) than 99 cents ($.99).

When we work just with whole numbers, we can easily tell which is greater. If we want to compare 435 and 439, we could simply write one number under the other so that the digits in the ones place line up.

$$\begin{array}{r} 435 \\ 439 \end{array}$$

Now, if we read both numbers from left to right we see that in both cases the digits in the hundreds place and in the tens place match. We do not have a matching in the ones place. Since the digits in the hundreds and the tens place match, all we need to do is compare the digits in the ones place, that is,

since $5 < 9$
then $435 < 439$.

To compare 99462 and 103578, we can do the same thing. Write the numbers with the ones places lined up:

$$\begin{array}{r} 99462 \\ 103578 \end{array}$$

We see at once that 103578 is greater than 99462 because there is a digit farther to the left in 103578 than in 99462.

Of course when we wrote the numbers we lined up the digits in the ones place. This automatically lined up the decimal points, even though they weren't written.

If we want to compare 9.29 and 10.2, we can write the numbers as before, lining up the digits in ones place, and therefore the decimal points, too.

$$\begin{array}{r} 9.29 \\ 10.2 \end{array}$$

We still look at the first digit on the left and say that 10.2 is greater than 9.29 (10.2 > 9.29). However, since we know that the places to the left of the decimal point are the whole number places, we can compare these two numbers mentally, because 10 is greater than 9.

When you compare decimals (numbers that are written with a decimal point), if there are whole number places you compare the whole numbers.

To compare numbers that are less than 1, or numbers in which the whole numbers are the same, we will use the same method as before. Numbers that are less than one are written to the right of the decimal point. Digits to the right of the decimal point are called the decimal places.

Class Discussion

Which of these two numbers is greater, .123 or .1229 ?

1. Before you compare any two decimals each decimal should have the same number of decimal places.

   (a) How many decimal places does .123 have? ____________

   (b) How many decimal places does .1229 have? ____________

   (c) If we "tack on" a zero to .123, we have added $\frac{0}{10000}$ to the number .123. Does this change its value? __________

2. To the right, we have rewritten .123 as .1230 and then written .1229 directly below it, making sure the decimal points line up.

   .1 2 3 0
   .1 2 2 9

   (a) Do the tenths match? __________

   (b) Do the hundredths match? __________

   (c) Do the digits in the thousandths place match? __________

(d) Which is greater, 3 or 2 ? ________

(e) Which is greater, .123 or .1229 ? ________

To compare two decimals:

(a) "Tack on" enough zeros so that both decimals have the same number of <u>decimal places</u>.

(b) Write one above the other, being sure that the decimal points line up.

(c) Starting at the decimal point, compare the digits in each place until you come to a place where the digits <u>do not</u> match.

(d) The decimal with the greatest digit in the unmatched place is the greater number.

<u>Example</u>. Which is greater, .001 or .0009 ?

Step (a): .0010 , .0009 ("Tacking on" the zero.)

Step (b): .0010
.0009 (Lining up the decimal point.)

Step (c): .0010
.0009 (Coming from the decimal point to a place where the digits <u>do not</u> match.)

Step (d): as $0 < 1$
then $.0009 < .001$

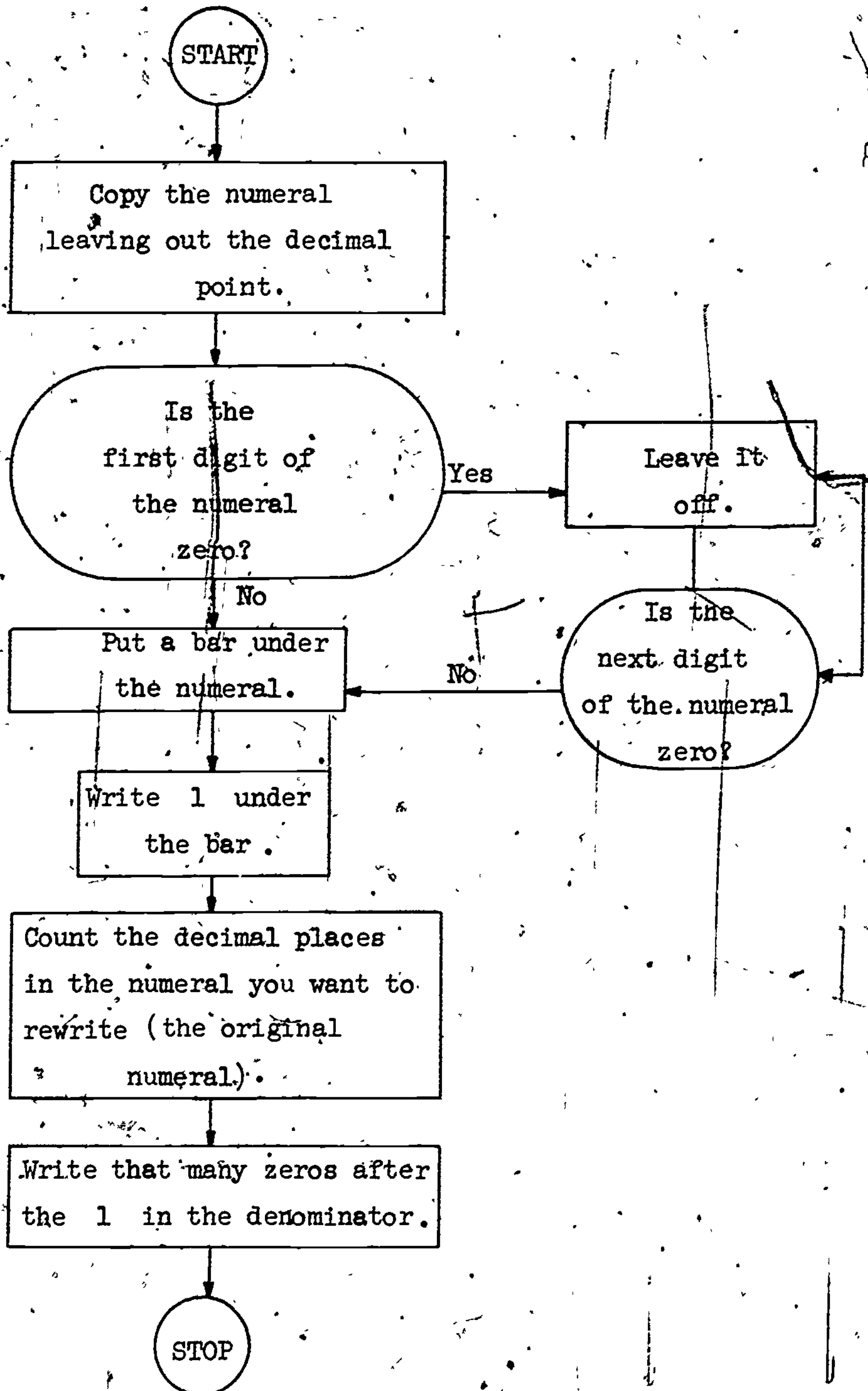
Exercises

Compare these decimal numbers. Show which is smaller by putting either < or > between them.

1. .07 _____ .0096
2. .5001 _____ .0999
3. .0001 _____ .0091
4. .0286 _____ .00989
5. .397 _____ .0309
6. .00001 _____ .000001
7. 2.03 _____ 2.079
8. 43.07 _____ 8.065
9. 593.76 _____ 593.761
10. 7.605 _____ 7.6005

Writing Decimals as Fractions

You know that .1 is the same number as $\frac{1}{10}$ and .03 is the same number as $\frac{3}{100}$. Sometimes you will need to rewrite other decimals as fractions. This flow chart will help you.



(Sometimes the fraction can be simplified.)

To rewrite .877 as a fraction, the steps would look like this:

.877 $\boxed{877}$ $\boxed{\underline{877}}$ $\boxed{\frac{877}{1}}$ $\boxed{\frac{877}{1000}}$

To rewrite .0063, the steps would look like this:

.0063 $\boxed{\cancel{0}\cancel{0}63}$ $\boxed{\underline{63}}$ $\boxed{\frac{63}{1}}$ $\boxed{\frac{63}{10000}}$

To rewrite 49.1, the steps would look like this:

49.1 $\boxed{491}$ $\boxed{\underline{491}}$ $\boxed{\frac{491}{1}}$ $\boxed{\frac{491}{10}}$

Exercises

Write these decimals as fractions. Use the flow chart if you need it.

1. .09 =	6. 4.019 =
2. 1.1 =	7. .53 =
3. .047 =	8. .6057 =
4. 45.81 =	9. .00009 =
5. .763 =	10. 100.0007 =

Division with Decimals

We have said over and over that the decimal point separates the whole part of a number from the part that is less than one. Numbers like 6 , or 118 , or 49 , are whole numbers. In 6, the six is in the ones place, and there are no tenths or hundredths or any other part less than one. Sometimes it is useful to write 6 like this: 6.00 . (This is like being able to write \$6 as \$6.00 .)

We can write any whole number and put a decimal point after it. Then, if we want to, we can write as many zeros as we want without changing the number at all.

Here are some examples:

2 and 2.000 have the same value, because 2.000 means 2 and $\frac{0}{10}$ and $\frac{0}{100}$ and $\frac{0}{1000}$.

5 and 5.0 have the same value, because 5.0 means 5 and $\frac{0}{10}$.

35 and 35.00 have the same value, because 35.00 means 35 and $\frac{0}{10}$ and $\frac{0}{100}$.

Class Discussion

Now we're ready to see how to find decimal names for rational numbers. Divide 2000.0 by 5 . What do you get? ________

Are $\frac{2000.0}{5}$ and 400.0 different names for the same number? ____

Divide 200.0 by 5 . What do you get? ________ . Are $\frac{200.0}{5}$ and 40.0 different names for the same number? ________

Divide 20.0 by 5 . What do you get? ________ Are $\frac{20.0}{5}$ and 4.0 different names for the same number? ________

Here is a list of what you did.

$\frac{2000.0}{5}$ ⟵ another name for ⟶ 400.0

$\frac{200.0}{5}$ ⟵ another name for ⟶ 40.0

$\frac{20.0}{5}$ ⟵ another name for ⟶ 4.0

What happened to the decimal point in the <u>numerator</u> on the left?

____

What did this do to the decimal point in the numbers on the right?

____

You can move the decimal in the numerator to the left once more and divide 2.0 by 5 . The answer is ____________ . The decimal point moved one place to the left in the answer.

$$\frac{2.0}{5} = \frac{2}{5} = \underline{\qquad}$$

Here is another example. Fill the blanks.

$\frac{3000.0}{6}$ = 500.0

$\frac{300.0}{6}$ = ________

$\frac{30.0}{6}$ = ________

$\frac{3}{6}$ = ________

If you <u>divide the numerator of a fraction by the denominator</u>, you get a decimal name for the same number.

Exercises

Find decimal names for these fractions. (There is space below each problem for you to do your division.)

Fraction name		Decimal name
1. (a) $\frac{40}{5}$	=	________
(b) $\frac{4}{5}$	=	________
2. (a) $\frac{300}{4}$	=	________
(b) $\frac{30}{4}$	=	________
(c) $\frac{3}{4}$	=	________

3. (a) $\frac{10}{2}$ = ____________________

(b) $\frac{1}{2}$ = ____________________

4. (a) $\frac{100}{4}$ = ____________________

(b) $\frac{10}{4}$ = ____________________

(c) $\frac{1}{4}$ = ____________________

Writing Fractions as Decimals

In Section 10-7, when you wrote decimals in fraction form, all the digits stayed the same except for the power of ten under the bar (and the zeros you sometimes left off).

$$.09 = \frac{9}{100}, \quad 3.47 = \frac{347}{100}, \quad 289.9 = \frac{2899}{10}, \quad \text{etc.}$$

In Section 10-8, however, you found, by dividing, that $\frac{4}{5} = .8$ and $\frac{3}{4} = .75$. The digits were different in the two names for the same number.

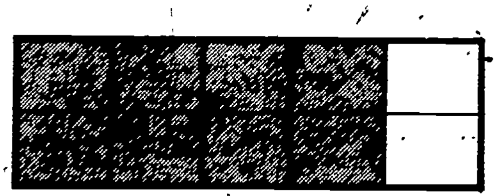
This is not unusual, of course. The name 7×7 does not <u>look</u> like 49, yet we know they name the same number. We can make sure by drawing 7 rows of dots with 7 dots in each row and counting to be certain there are 49 in all.

How can we make sure that $\frac{4}{5}$ and $.8$ are the same number?

One way is to write $.8$ as a fraction and see if the two fractions are equal.

$$.8 = \frac{8}{10} \quad \text{and} \quad \frac{8}{10} = \frac{4}{5} \times \frac{2}{2}$$

Or we could use a unit region divided into fifths and then into tenths.



Again we see that $\frac{4}{5} = \frac{8}{10}$.

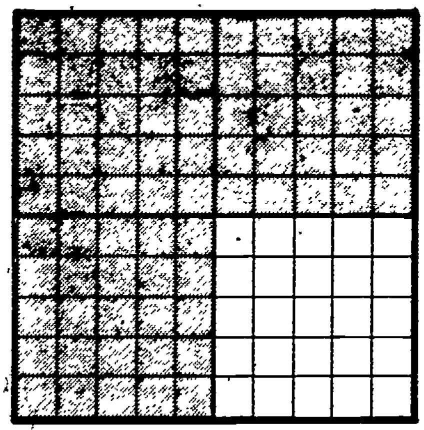
To be sure $.75 = \frac{3}{4}$, we can change the decimal to a fraction and see that

$$.75 = \frac{75}{100}$$

and $$\frac{75}{100} = \frac{3}{4} \times \frac{25}{25}.$$

We can use a unit region divided into fourths and then into hundredths.

If we shade $\frac{3}{4}$ of the unit, we find that what we shaded is exactly $\frac{75}{100}$ of it.



We can write <u>any</u> rational number in either fraction or decimal form. If the rational number is a whole number, we can write it as a decimal just by putting a decimal point after it and writing as many zeros as we like after the decimal point.

$$45 = 45.0$$

We can write it as a fraction in many ways: $45 = \frac{45}{1}$ or $\frac{90}{2}$ or $\frac{135}{3}$, etc.

Class Discussion

A fraction with a power of 10 as the denominator is easy to write as a decimal. All we do is copy the numerator, count the zeros in the denominator, and count that many places <u>from right to left</u> before putting in the decimal point.

If we want to change $\frac{13}{10}$ to a decimal, we copy the numerator, _____. How many zeros are in the denominator? _________ How many decimal places will there be in the decimal? _________ Write $\frac{13}{10}$ as a decimal. _________

If we want to change $\frac{19}{1000}$ to a decimal, we copy the numerator, ______. How many zeros are in the denominator? ______. How many decimal places will there be in the decimal? __________. When you count from right to left you find you need one more digit before you write the decimal point. Write a zero. So $\frac{19}{1000} = .019$.

Another way to change a fraction to a decimal can be used whether the denominator is a power of 10 or not.

Since a fraction is just a division expression, we can simply divide the numerator by the denominator.

$\frac{13}{10}$ means ______ divided by 10 .

Write the problem this way: $10\overline{)13}$

Can you divide? ______

Write the 1 in the answer in the ones place, and put a decimal point after it, because the 1 represents a whole number.

Write the left-over 3 to the right of the 3 in the dividend.

$$\begin{array}{r} 1. \\ 10\overline{)13^3} \end{array}$$

This left-over 3 is 3 ones. Put a decimal point after 13 and write a zero.

$$\begin{array}{r} 1. \\ 10\overline{)13.^30} \end{array}$$

Think of the left-over 3 as _____ tenths. Can you divide? ______ 30 tenths divided by 10 is ______ tenths. Write 3 in the tenths place of your answer.

$$\frac{13}{10} = \underline{\qquad\qquad}$$

Write $\frac{19}{1000}$ this way. $1000\overline{)19}$

Can you divide 19 by 1000 ? ______ Put a decimal point after 19 and just above put a decimal point in your answer. You know your answer will not have any whole number places.

Write a zero in the dividend.

19 = _____ tenths

Can you divide 190 tenths by 1000? _____

Write 0 in tenths place in your answer.

Put another 0 in the dividend.

19 = _____ hundredths

Can you divide? _____

Write 1 in hundredths place in your answer.

$$\begin{array}{r} .01 \\ 1000\overline{)19.00} \\ -10\ 00 \\ \hline 9\ 00 \end{array}$$

Write another 0 in the dividend.

900 hundredths = 9000 thousandths

$$\begin{array}{r} .019 \\ 1000\overline{)19.000} \\ -10\ 00 \\ \hline 9\ 000 \\ -9\ 000 \\ \hline \end{array}$$

$\frac{19}{1000} = .019$

You won't want to use the division method for finding decimal names of fractions that have a power of 10 in the denominator, but it is a good method for other fractions.

In changing fractions to decimals, remember to:

(1) Put a decimal point after the whole number in the dividend.

(2) Put a decimal point in the answer just above the point in the dividend.

(3) When you can't divide, put a zero in the correct place in the answer.

(4) Use just as many zeros as necessary after the decimal point in the dividend.

Here is an example of the steps in writing $\frac{1}{16}$ as a decimal.

$$\begin{array}{r} . \\ 16\overline{)1.} \end{array}$$

$$\begin{array}{r} .0 \\ 16\overline{)1.0} \end{array}$$

$$\begin{array}{r} .06 \\ 16\overline{)1.00} \\ -96 \\ \hline 4 \end{array}$$

$$\begin{array}{r} .062 \\ 16\overline{)1.000} \\ -96 \\ \hline 40 \\ -32 \\ \hline 8 \end{array}$$

$$\begin{array}{r} .0625 \\ 16\overline{)1.0000} \\ -96 \\ \hline 40 \\ -32 \\ \hline 80 \\ -80 \\ \hline \end{array}$$

None of the problems in the following exercises is this long!

Exercises

1. Divide to find the decimal name for these.

(a) $\frac{1}{4}$ = ________

(b) $\frac{3}{8}$ = ________

(c) $\frac{2}{5}$ = ________

(d) $\frac{9}{10}$ = ________

(e) $\frac{7}{8}$ = ________

(f) $\frac{4}{5}$ = ________

(g) $\frac{5}{8}$ = ________

(h) $\frac{3}{10}$ = ________

You have to be careful when you find the decimal name for numbers greater than one. Look:

$$\frac{3}{2} > 1$$

Because this is true, there will be a digit to the left of the decimal point.

Divide as usual: $2\overline{)3}$

You can divide: $\begin{array}{r} 1 \\ 2\overline{)3^{1}} \end{array}$

but you have a remainder. Put a decimal point in the dividend and a decimal point above it in the answer.

$$\begin{array}{r} 1. \\ 2\overline{)3.^{1}0} \end{array}$$

Divide again. $\begin{array}{r} 1.\,5 \\ 2\overline{)3.^{1}0} \end{array}$

2. Divide to find the decimal name for these.

(a) $\frac{5}{4}$ = ______ (f) $\frac{3}{20}$ = ______

(b) $\frac{5}{2}$ = ______ (g) $\frac{4}{25}$ = ______

(c) $\frac{7}{5}$ = ______ (h) $\frac{19}{20}$ = ______

(d) $\frac{9}{4}$ = ______ (i) $\frac{9}{5}$ = ______

(e) $\frac{1}{5}$ = ______ (j) $\frac{9}{8}$ = ______

Repeating Decimals

So far, in changing fractions to decimals, you have found that after dividing to two or three decimal places you could stop, because there was no remainder. In the example which was worked for you, $\frac{1}{16} = .0625$, it was necessary to write four zeros in the dividend before there was no remainder.

For some fractions, there is "no place to stop". These are called repeating decimals.

Class Discussion

We will rewrite $\frac{1}{3}$ as a decimal.

$$3\overline{)1.0}$$

When you divide 10 tenths by 3, you get ________ with a remainder of ______ tenth.

$$\begin{array}{r} .3 \\ 3\overline{)1.0\,0} \end{array}$$

When you divide 10 hundredths by 3, you get ________ with a remainder of ______ hundredth.

$$\begin{array}{r} .3\;3 \\ 3\overline{)1.0\,0\,0} \end{array}$$

When you divide 10 thousandths by 3, you get ________ with a remainder of ______ ten-thousandth.

It is clear that since you keep on dividing 10 of something by 3, you will always get ________ with a remainder of ________. The digit ____, in the decimal, will be repeated no matter how many decimal places you use. To show that this happens, we put a bar over the digit that is repeated: $\frac{1}{3} = .\overline{3}$.

We will rewrite $\frac{1}{11}$ as a decimal.

$$11\overline{)1.00}$$

Can you divide? ________

Write 0 in tenths place. You have 10 tenths left over, and 10 tenths = ______ hundredths.

$$11\overline{)1.0^{10}0}$$ with quotient $.0$

Can you divide? ________

$$11\overline{)1.0^{10}0^{1}0}$$ with quotient $.0\,9$

The remainder is 1. The next digit in the answer is 0, and the one after that is 9 again. No matter how long you work you will get first a 0, then a ______. Since this is true, we write $\frac{1}{11} = .\overline{09}$. This tells us that

$$\frac{1}{11} = .090909090909090909 \text{ and so on.}$$

Here is a longer one. $\frac{3}{7} = ?$ $\quad 7\overline{)3.\qquad\qquad}$

Divide 3 by 7 and keep on writing zeros in the dividend as long as you need them. Your first left-over is, of course, 3 ones. Then you will get left-overs of 2, 6, 4, 5, and 1. The next left-over is 3, the number of ones you started with, so you know the digits in the quotient will now repeat. Write the decimal numeral for $\frac{3}{7}$. ________

Try this one: $\frac{5}{3} = ?$

$\frac{5}{3}$ is greater than 1. Your answer has a 1 in the whole number part. The "left-over" 2 repeats each time you divide, so $\frac{5}{3} = 1.\overline{6}$.

When you divide, you can stop as soon as the "left-over" is the same as the number of left-over ones.

Sometimes you want to know before you start whether you can write a fraction as a <u>terminating decimal</u> (one that "stops") or whether you'll get a repeating decimal.

Look at the denominators of these fractions that give terminating decimals:

$$\frac{1}{16}, \frac{3}{8}, \frac{3}{4}, \frac{1}{25}, \frac{7}{20}, \frac{11}{50}, \frac{5}{64}.$$

If you find the prime factorization of any of these denominators, you will see that they have no prime factors except 2 or 5, and <u>2 and 5 are the only prime factors of any power of 10</u>.

Here are some fractions that give repeating decimals:

$$\frac{5}{18}, \frac{7}{6}, \frac{3}{11}, \frac{1}{13}, \frac{2}{15}, \frac{5}{24}.$$

Each denominator has some prime factor <u>other</u> than 2 and 5.

Exercises

Write the decimal numeral for each of the following fractions. (See if you can tell, before you start, which ones will repeat.)

1. $\frac{5}{2} =$	6. $\frac{5}{6} =$
2. $\frac{2}{3} =$	7. $\frac{1}{12} =$
3. $\frac{4}{9} =$	8. $\frac{1}{6} =$
4. $\frac{7}{4} =$	9. $\frac{1}{8} =$
5. $\frac{7}{8} =$	10. $\frac{8}{9} =$

Multiplication with Decimals

In order to multiply using decimals, you need to be able to multiply whole numbers. The problem 4×37 could be worked by adding four 37's, but there are shorter ways.

Because $37 = 30 + 7$, you can first multiply 4×30 and then multiply 4×7 and add your answers.

$$\begin{array}{r} 30 \\ \times\, 4 \\ \hline 120 \end{array} \quad \text{and} \quad \begin{array}{r} 7 \\ \times\, 4 \\ \hline 28 \end{array} \quad \text{so} \quad 4 \times 37 = 120 + 28 \text{ or } 148$$

We can write this:

$$\begin{array}{rl} 37 & \\ \times\, 4 & \\ \hline 120 & (4 \times 30) \\ 28 & (4 \times 7) \\ \hline 148 & \end{array}$$

or this:

$$\begin{array}{rl} 37 & \\ \times\, 4 & \\ \hline 28 & (4 \times 7) \\ 120 & (4 \times 30) \\ \hline 148 & \end{array}$$

To save time and writing, however, we can start at the right, think $4 \times 7 = 28$, and since the 2 means 2 tens, we remember 2 until we have found what 4×30 is. We then add the 2 tens to the 12 tens in 120. Sometimes people write the 2 above the tens place in 37 as a reminder, like this:

$$\begin{array}{r} {}^{2}37 \\ \times\, 4 \\ \hline 148 \end{array}$$

We can multiply 3×235 like this:

$$\begin{array}{rl} 235 & \\ \underline{\times 3} & \\ 15 & (3 \times 5) \\ 90 & (3 \times 30) \\ \underline{600} & (3 \times 200) \\ 705 & \end{array}$$

or like this:

$$\begin{array}{r} {}^{1\,1}\\[-4pt] 235 \\ \underline{\times 3} \\ 705 \end{array}$$

Exercises

Multiply. Use the method that is easiest for you.

1. $\begin{array}{r} 47 \\ \underline{4} \end{array}$
2. $\begin{array}{r} 68 \\ \underline{9} \end{array}$
3. $\begin{array}{r} 63 \\ \underline{7} \end{array}$
4. $\begin{array}{r} 41 \\ \underline{5} \end{array}$
5. $\begin{array}{r} 368 \\ \underline{7} \end{array}$
6. $\begin{array}{r} 264 \\ \underline{6} \end{array}$
7. $\begin{array}{r} 473 \\ \underline{8} \end{array}$
8. $\begin{array}{r} 987 \\ \underline{5} \end{array}$

Multiplication with Larger Numbers

In a problem like 34×76, we know that $34 = 30 + 4$, so 76 (which is $70 + 6$) must be multiplied both by 30 and by 4. Again, it doesn't matter which we do first as long as we multiply 30×70, 4×70, 30×6, and 4×6 and find the sum of these products. Luckily, our place value system does a lot of the work for us. Here is one way to do this:

$$\begin{array}{rl} 76 & \\ \underline{\times 34} & \\ 24 & (4 \times 6) \\ 280 & (4 \times 70) \\ 180 & (30 \times 6) \\ \underline{2100} & (30 \times 70) \\ 2584 & \end{array}$$

Some people write less. They solve 4×76 by multiplying 4×6 and then remembering that the 2 tens in 24 must be added to the product of 4×70. Their first step would look like this:

$$\begin{array}{r} 76 \\ \underline{\times 34} \\ 304 \end{array}$$

They then write a 0 under the 4 (because they are going to multiply by 30) and then find 3×76. The completed problem looks like this:

$$\begin{array}{r} 76 \\ \underline{\times 34} \\ 304 \\ \underline{2280} \\ 2584 \end{array}$$

Exercises

Multiply in any way you like.

1. $\begin{array}{r} 67 \\ \underline{24} \end{array}$

2. $\begin{array}{r} 45 \\ \underline{63} \end{array}$

3. $\begin{array}{r} 97 \\ \underline{69} \end{array}$

4. $\begin{array}{r} 32 \\ \underline{78} \end{array}$

5. $\begin{array}{r} 346 \\ \underline{67} \end{array}$

6. $\begin{array}{r} 473 \\ \underline{24} \end{array}$

Decimals in Multiplication

In order to understand how to multiply using decimals, we think of how decimals can be written as fractions. When we multiply $1.3 \times .5$, for instance, we are multiplying $\frac{13}{10} \times \frac{5}{10}$.

Class Discussion

$$\frac{13}{10} \times \frac{5}{10} = \frac{13 \times 5}{10 \times 10} \quad \begin{pmatrix} 13 \times 5 = \underline{\qquad} \\ 10 \times 10 = \underline{\qquad} \end{pmatrix}$$

So $\frac{13}{10} \times \frac{5}{10} =$ ______

Rewrite the product as a decimal. ______

How many decimal places are in 1.3 ? ______

How many decimal places are in .5 ? ______

How many decimal places are in the product? ______

To multiply 2.56×1.5 we can rewrite both numbers as fractions.

2.56 = ______ and 1.5 = ______.

$$\frac{256}{100} \times \frac{15}{10} = \frac{256 \times 15}{100 \times 10}$$

$256 \times 15 =$ ______ So $\frac{256}{100} \times \frac{15}{10} =$ ______

$100 \times 10 =$ ______ Write this as a decimal ______

How many decimal places are in 2.56 ? ______

How many decimal places are in 1.5 ? ______

How many decimal places are in 3.840? ______

When we multiply powers of 10, we just count all the zeros in the numbers we are multiplying and put that many ______ in the answer. The number of decimal places in a number like 2.56 just shows how many ______ are in the denominator of the fraction

$\frac{256}{100}$. Therefore when we multiply decimals we just count all the decimal places in the numbers we are multiplying and put that many _______ places in the answer.

To multiply $.36 \times .2$, we write the problem:

$$\begin{array}{r} .36 \\ \underline{\times\ .2} \end{array}$$

Notice that we don't have to keep the decimal points lined up when we multiply. If we rewrite the problem in fraction form,

$\frac{36}{100} \times \frac{2}{10}$, we would multiply whole numbers, 36×2 and 100×10.

Now we multiply <u>as if</u> $.36$ and $.2$ were whole numbers, then count the decimal places (two in $.36$ and one in $.2$) and put the decimal point in the answer so that there are three decimal places. But $.36 \times .2$ is 72, and we must have 3 decimal places, so we put a 0 before the 7.

$$\begin{array}{r} .36 \\ \underline{\times\ .2} \\ .072 \end{array}$$

$.36 \times .2 = .072$

Exercises

1. Multiply. Use your tables when you can.

   (a) $.09 \times .09 =$ __________

   (b) $2.5 \times .0025 =$ __________

   (c) $1.2 \times 120 =$ __________

   (d) $1.135 \times 2.2 =$ __________

   (e) $7.48 \times .12 =$ __________

   (f) $.0762 \times .11 =$ __________

   (g) $4 \times .25 =$ __________

   (h) $.5 \times .25 =$ __________

   (i) $2.7 \times .15 =$ __________

2. Multiply.

   (a) $52 \times .03 =$ __________

   (b) $59.8 \times 2.1 =$ __________

   (c) $4.6 \times 11.0 =$ __________

   (d) $932 \times .002 =$ __________

3. Multiply.

   (a) $.7 \times .01 =$ __________

   (b) $498 \times .1 =$ __________

   (c) $.32 \times .01 =$ __________

   (d) $.0068 \times 10 =$ __________

   (e) $2.75 \times .01 =$ __________

   (f) $32.596 \times .1 =$ __________

   (g) $.993 \times 1000 =$ __________

Decimal Divisors

Most people think it is easier to divide by a whole number than by a fraction. Which looks easiest,

$$\frac{4}{2} = ? \quad \text{or} \quad \frac{4}{2.0} = ? \quad \text{or} \quad \frac{4}{\frac{6}{3}} = ?$$

You can see that they all are the same number, 2, but the first one looks easier than the others.

This is why many people rename a number to find the answer in a problem like $.5\overline{)20}$.

First we find another way of writing the problem. You have often rewritten numbers like $\frac{1}{5}$ this way: $5\overline{)1.}$. This time we will write the dividend in our problem (20) as the numerator of a fraction, and the divisor (.5) as the denominator, just the opposite of what we did before.

We write $.5\overline{)20}$ as $\frac{20}{.5}$.

Class Discussion

We want to multiply the number $\frac{20}{.5}$ by some name for 1 so that there will not be any decimal places in the denominator. If the denominator were 5 instead of .5 we could divide in the usual way.

What we want to do is move the decimal point one place to the right. If we multiply .5 by ______ we get 5 . Now we have the helpful name for 1: $\frac{10}{10}$.

$$\frac{20}{.5} \times \frac{10}{10} = \frac{200.}{5.0} \quad \text{or} \quad \frac{200}{5}$$

Write the problem like this: $5\overline{)200}$ and divide as usual.

If 20 divided by .5 is 40 , then $40 \times .5 = 20$. Do the multiplication to see that this is correct.

$$\begin{array}{r} 40 \\ \underline{\times .5} \end{array}$$

To divide 75 by .25 , we can first write

$$\frac{75}{.25}$$

Since there are two decimal places in .25 , we will have to multiply it by ______ to get the whole number, 25 . We choose $\frac{100}{100}$ as our name for 1 .

$$\frac{75}{.25} \times \frac{100}{100} = \frac{\quad}{25.}$$

Now divide as usual. $25\overline{)7500}$

Check your answer by multiplying: $300 \times .25 =$ ________

In the problem 6.8 divided by .02 , you will have to multiply .02 by ______ to get the whole number 2 . The name we choose to multiply by is ______.

$$\frac{6.8}{.02} \times \frac{100}{100} = \underline{\qquad}$$

When you are sure you understand this, you can save time in division problems like $.02\overline{)6.8}$.

Count the decimal places in the divisor (.02) to find out what number you will multiply both numbers by. To move the decimal point two places to the right, you multiply both numbers by a hundred. Simply cross out both decimal points and move them two places to the right.

$$\cancel{.}02.\overline{)6\cancel{.}80.}$$

Then divide as usual.

In this class discussion you saw that you can write a new name for a fraction that has a decimal denominator. The way this is done is to multiply the fraction by one. The name for one uses the same power of ten for both the numerator and denominator. Once you write the new name that has a whole number for the denominator, you know how to divide.

Exercises

Follow these steps to work each problem below.

First: Find out what power of 10 you must multiply the denominator by to make it a whole number.

Second: Write a name for 1 using that power of 10 .

Third: Multiply by 1 .

Fourth: Divide the numerator of the new fraction by the denominator of the new fraction.

Fifth: Write your answer in the blank at the right.

Last: Check your answer by multiplying it by the denominator of the fraction you started with.

Example: $\frac{3}{.25} \times \frac{100}{100} = \frac{300}{25}$; $25\overline{)300}$ = 12 (25, 50, 50) ______12______

1. (a) $\frac{4}{.5}$ (a) ________

(b) $\frac{5}{.8}$ (b) ________

(c) $\frac{1}{.2}$ (c) ________

(d) $\frac{4}{2.5}$ (d) ________

(e) $\frac{6}{.05}$ (e) ________

2. Be careful of the decimal point in these.

Example. $\frac{.65}{.5} \times \frac{10}{10} = \frac{6.5}{5}$ and $5\overline{)6.5}^{\,1.3}$ 1.3

(a) $\frac{.75}{.3}$ (a) ________

(b) $\frac{.125}{.5}$ (b) ________

(c) $\frac{8}{.025}$ (c) ________

(d) $\frac{.6}{.02}$ (d) ________

(e) $\frac{.49}{.7}$ (e) ________

(f) $\frac{1.28}{.8}$ (f) ________

3. Divide. Remember: To get rid of the decimal places in the divisor you must multiply __both__ numbers by the __same__ power of 10 .

(a) $.6\overline{)3.66}$

(b) $.09\overline{)891}$

(c) $1.2\overline{).36}$

(d) $2.5\overline{)10}$

(e) $.14\overline{)9.8}$

(f) $.23\overline{).115}$

(g) $1.8\overline{)7.2}$

(h) $15.0\overline{)900}$

Addition and Subtraction with Decimals

When you add and subtract whole numbers, you are careful to write the problem down so that you add ones to ones, tens to tens, and so on. You have done this with whole numbers by making sure the ones digits were lined up in a column. To add 33 + 9 + 148 + 22, you write:

```
 33
  9
148
 22
```

When you have to add or subtract amounts of money, you line up the decimal points to make sure you add dollars to dollars, dimes to dimes and pennies to pennies. (You wouldn't write 1 dollar and 1 dime in a column and add! 1 and 1 are 2, and you'd have to ask, "Two what?")

Class Discussion

When you write whole numbers in a column, you line up the digits in __________ place. You can always think of a decimal point as being just to the ______ of the ones place, so you are automatically lining up the ________________.

To add decimal numbers, you must be careful to keep the decimal points lined up, one directly beneath the other. It is often helpful to make sure all the numbers you are to add have the same number of decimal places. If you want to add 3.8 and .009, you can't move the digit 9 to the left, but you can put zeros after the 8, because 3.800 is the same number as 3.8. You will write your problem like this:

```
3.800
 .009
```

The decimal point in your answer must be lined up, too, so that the sum has three decimal places: 3.809 .

To subtract, you set up the problem the same way.

Example. 4.37 - 3.259

4.37 has only two decimal places and 3.259 has three. Write 4.37 as 4.370 and set up the problem like this:

$$\begin{array}{r} 4.370 \\ \underline{3.259} \end{array}$$

Now subtract in the usual way. The decimal point in the answer goes directly below the other decimal points.

$$\begin{array}{r} 4.370 \\ \underline{3.259} \\ 1.111 \end{array}$$

Exercises

1. Add. Be sure to write enough zeros at the right to keep the decimal points lined up. Problem (a) is done for you.

(a) .7 + .84 = 1.54

$$\begin{array}{r} .70 \\ \underline{+\ .84} \\ 1.54 \end{array}$$

(b) .719 + .382 = ____________

(c) .853 + .16 = ____________

(d) .625 + .55 = ____________

(e) 1.002 + .0102 = ____________

(f) $.5 + .125 =$ ____________

(g) $.407 + .32 + .076 =$ ____________

(h) $2.314 + .6 + .72 =$ ____________

(i) $1.05 + .075 + 21.5 =$ ____________

2. Subtract. Problem (a) is done for you.

(a) $4.6 - 2.36 =$ 2.24

$$\begin{array}{r} 4.60 \\ -2.36 \\ \hline 2.24 \end{array}$$

(b) $48.134 - 16.78 =$ ____________

(c) $.807 - .235 =$ ____________

(d) $.64 - .476 =$ ____________

(e) $1.216 - .438 =$ ____________

(f) $14.273 - 4.39 =$ ____________

(g) $7.28 - 5.542 =$ ____________

(h) $1.005 - .0326 =$ ____________

Summary

In this chapter you have learned that any rational number can be named either in fraction form or in decimal form. Sometimes it is easier to work with fractions and sometimes it is easier to work with decimals. If you know how to do both, you can solve the problem the easier way.

Class Discussion

Suppose you are told to multiply $64 \times .125$. You can do this:

$$\begin{array}{r} .125 \\ \underline{\times\ 64} \\ 500 \\ \underline{7500} \\ 8.000 \end{array}$$

but if you know that $.125 = \frac{1}{8}$, you can do this:

$$\frac{1}{8} \times 64 = \frac{64}{8}$$

$$= 8$$

Suppose you have to add $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$. One way to do it is to think $\frac{7}{8} = .875$, $\frac{3}{4} =$.____, and $\frac{1}{2} =$.____.

$$\begin{array}{r} .875 \\ .750 \\ \underline{.500} \\ 2.125 \end{array}$$

$2.125 = 2\frac{1}{8}$

In order to use whichever form is easiest, you must be able to find decimal names for fractions and fraction names for decimals.

Remember that fractions are division expressions, so to change $\frac{1}{4}$ to a decimal, you divide 1 by _____ .

$$4\overline{)1.00}\quad .25$$

It is useful just to <u>remember</u> names that go together for some of the commonly used fractions. If you know that $\frac{1}{8} = .125$, for instance, then to find $\frac{2}{8}$ (which is also $\frac{1}{4}$) you can just multiply $.125 \times 2$ and get $.250$. $\frac{3}{8} = 3 \times .125$, $\frac{5}{8} = 5 \times .125$, and $\frac{7}{8} = 7 \times .125$. (Do you see an easy way to find $\frac{1}{16}$, now? $\frac{1}{16}$ is $\frac{1}{2} \times \frac{1}{8}$, so $\frac{1}{16} = \frac{.125}{2}$, or $.0625$.)

To find fraction names for decimals that do not repeat, you first rewrite the decimal as a fraction with the denominator a power of 10 . You remember that there are as many zeros in the denominator as there are decimal places.

$$.125 = \frac{125}{1000}$$

If you divide both the numerator and the denominator by 5 as long as you can, you can simplify the fraction.

$$.125 = \frac{125}{1000}$$

$$= \frac{25}{200}$$

$$= \frac{5}{40}$$

$$= \frac{1}{8}$$

The most important thing for you to remember about decimals is that the decimal point separates the _______________ part of the decimal from the part that is less than 1. Digits to the _________ of the decimal point represent whole numbers, and the farther they are from the point, the ________ the number they represent. In the number 6,000,000, the 6 means six <u>million</u>. Digits to the ________ of the decimal point represent numbers that are <u>less</u> than 1 , and the farther they are from the point the <u>smaller</u> they get. It takes 1000 of these: <u>.006</u> to make <u>6</u> .

With many problems, you can find out whether your answer is <u>reasonable</u> in order to check up on where you put the decimal point.

In 41.3 + 2.069 , the whole numbers are 41 and ________. Your answer should be <u>about</u> 43 , because 41.3 is just a little larger than 41 and 2.069 is just a little larger than 2 .

In 60.8 - 1.79 , the whole numbers are _____ and _____, so your answer should be <u>about</u> _____ because 60 - 1 is _____.

In 5.63 × 2.1 , the whole numbers are _____ and _____. Your answer should be <u>about</u> _____, because 5 × 2 is _____.

In $\frac{25.06}{5.1}$, your answer should be <u>about</u> _____ because _____ divided by _____ is _____.

Here are a few rules to help you with the exercises.

1. When you add or subtract or compare decimals, it's important to keep the decimal points lined up.

2. When you multiply decimals, you multiply <u>as if</u> they were whole numbers. The answer has as many decimal places as there were all together in the problem. (Like the zeros in the denominators, remember?)

3. When you divide by a decimal, you first multiply both numbers by the power of 10 that has as many zeros as there are decimal places in the divisor.

4. To multiply by 10, move the decimal point one place to the right.

5. To divide by 10, move the decimal point one place to the left.

Exercises

1. Rename these fractions as decimals.

   (a) $\frac{3}{4} =$

   (b) $\frac{5}{8} =$

   (c) $\frac{2}{3} =$

   (d) $\frac{4}{5} =$

2. Rename these decimals as fractions. (Some of them you will need to simplify.)

   (a) .6

   (b) .12

   (c) .45

   (d) .13

   (e) 2.077

   (f) .0031

3. Write < or > in the blank to make these true.

(a) .0197 _____ .2

(b) 7.6 _____ 7.75

(c) 40.001 _____ 40.0011

(d) .987 _____ .9836

4. Add.

(a) 6.8 + 3.042 + 71 =

(b) 21.007 + 32.963 + .18 =

5. Subtract.

(a) 7.9 - 2.5 =

(b) 3.04 - 1.562 =

(c) 58.079 - 21.2 =

6. Multiply.

(a) 39.1 × .001 =

(b) 2.32 × 1000 =

(c) 42 × .01 =

7. Divide.

(a) $.09\overline{)81}$ (b) $2.1\overline{)6.3}$ (c) $1.8\overline{).36}$

Chapter 10. Decimals

Pre-Test Exercises

These exercises are like the problems you will have on the chapter test. If you don't know how to do them, read the section again. If you still don't understand, ask your teacher.

1. (Section 10-9) Divide.

(a) $\frac{498}{.10}$ = ____________

(b) $\frac{2}{1000}$ = ____________

(c) $\frac{468.2}{100}$ = ____________

2. (Section 10-11) Multiply.

(a) 5.8×100 = ____________

(b) $.022 \times 21$ = ____________

(c) $.25 \times 4.8$ = ____________

(d) 4.76×1000 = ____________

(e) $3 \times .125$ = ____________

3. (Section 10-13) Add.

(a) $2.9 + 3.07 + 29 + 1.004$ = ____________

(b) $1.25 + .046 + 3.87 + .9$ = ____________

4. (Section 10-13) Subtract.

(a) $3.45 - 2.15$ = ____________

(b) $8.1 - 3.125$ = ____________

5. (Section 10-12) Divide.

(a) $\frac{2.8}{.4}$ = ______________

(b) $\frac{26}{.02}$ = ______________

(c) $\frac{.56}{.8}$ = ______________

6. (Section 10-9)

Write the following as decimals.

	Decimal
(a) $\frac{1}{2}$	______
(b) $\frac{1}{4}$	______
(c) $\frac{73}{100}$	______
(d) $\frac{3}{2}$	______
(e) $\frac{3}{8}$	______

7. (Sections 10-7 and 10-14)

Write the following as fractions in simplest form.

(a) .35 = ______________   (c) .4 = ______________

(b) .625 = ______________   (d) .05 = ______________

8. (Section 10-11)
Fill the blanks.

(a) .25 × 60 is ____________

(b) .20 × 30 is ____________

(c) .05 × 535 is ____________

9. (Section 10-6)
Write < or > to show which number is greater.

(a) .039 ________ .0164

(b) .104 ________ .13

(c) .0051 ________ .00512

TEST

Chapter 10

1. Divide.

(a) $\frac{36.8}{100}$ = ____________

(b) $\frac{2.9}{1000}$ = ____________

(c) $\frac{.08}{10}$ = ____________

2. Multiply.

(a) .29 × 10 = ____________

(b) 1.8 × 2.9 = ____________

(c) .82 × 100 = ____________

(d) .15 × .23 = ____________

(e) 2.6 × 22 = ____________

3. Add.

(a) 64 + 2.5 + 3.147 + 16 = ____________

(b) 9.43 + .08 + 2.1 + 45.6 = ____________

4. Subtract.

(a) $76.04 - 7.1 =$ ____________

(b) $29.4 - 27.39 =$ ____________

5. Divide.

(a) $\frac{255}{.01} =$ ____________

(b) $\frac{.366}{.6} =$ ____________

(c) $\frac{3.2}{.2} =$ ____________

6. Write the following as decimals.

	Decimal
(a) $\frac{1}{4}$	________
(b) $\frac{3}{4}$	________
(c) $\frac{1}{8}$	________
(d) $\frac{1}{2}$	________
(e) $\frac{89}{100}$	________
(f) $\frac{1}{5}$	________
(g) $\frac{1}{10}$	________
(h) $\frac{5}{4}$	________

7. Write the following as fractions in simplest form.

(a) .45 = ______ (d) .2 = ______

(b) .375 = ______ (e) .04 = ______

(c) .8 = ______ (f) .15 = ______

8. Fill the blanks.

(a) .35 × 200 is ______

(b) .15 × 48 is ______

(c) 100 × 18 is ______

(d) .05 × 465 is ______

(e) .1 × 397 is ______

9. Write < or > to show which number is greater.

(a) .29 ______ .289

(b) .01604 ______ .016041

(c) .078 ______ .064

Check Your Memory: Self-Test

1. (Section 6-5.)

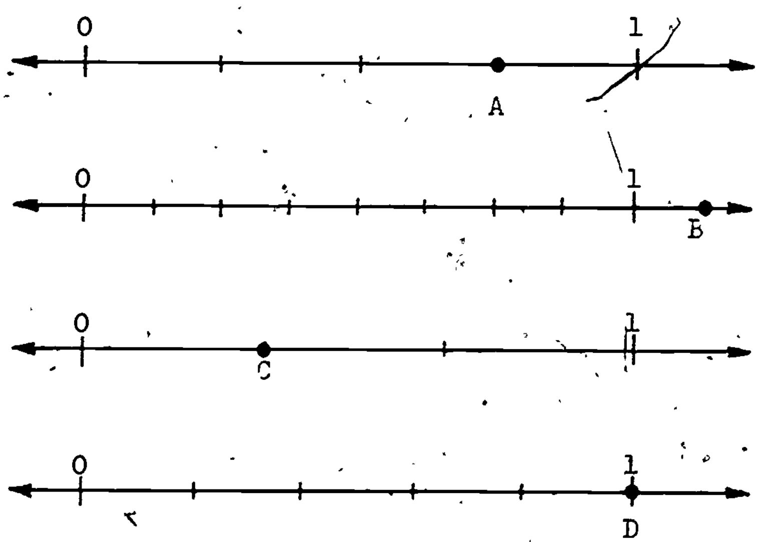
(a) $\frac{16}{2}$ = ________ and ________ $\times 2 = 16$

(b) $\frac{169}{13}$ = ________ and ________ $\times 13 = 169$

(c) $\frac{700}{25}$ = ________ and ________ $\times 25 = 700$

2. (Section 6-2.)

What rational number is represented at each point named?



A ________

B ________

C ________

D ________

3. (Section 6-2.)

Fill the blanks.

(a) $\frac{2}{5}$ = ________ $\times \frac{1}{5}$

(b) $\frac{8}{7} = 8 \times$ ________

(c) $\frac{3}{3}$ = ________ $\times \frac{1}{3}$

4. (Section 8-6.)

Fill the blanks so that the equation is equivalent to the one just before it.

(a) $2x + 5 = 17$

$2x =$ ______

$x =$ ______

(b) $7x - 10 = 4$

______ $= 14$

$x =$ ______

(c) $\frac{3}{5}x + 7 = 22$

$\frac{3}{5}x =$ ______

$x =$ ______

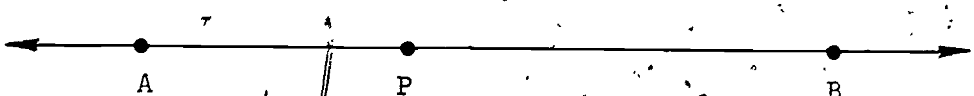
(d) $\frac{4}{3}x + {}^{-}\frac{2}{5} = \frac{3}{5}$

______ $= 1$

$x =$ ______

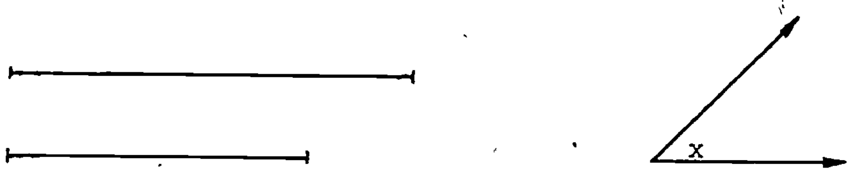
5. (Section 9-10.)

Construct a line perpendicular to $\overleftrightarrow{AB}$ at point P.



6. (Section 9-5.)

Draw a triangle with two sides and the angle they form congruent to the segments and angle below.



Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1. (a) 8, 8

   (b) 13, 13

   (c) 28, 28

2. A $\frac{3}{4}$

   B $\frac{9}{8}$

   C $\frac{1}{3}$

   D $\frac{5}{5}$

3. (a) 2

   (b) $\frac{1}{7}$

   (c) 3

4. (a) $2x + 5 = 17$

   $2x = 12$

   $x = 6$

   (b) $7x - 10 = 4$

   $7x = 14$

   $x = 2$

   (c) $\frac{3}{5}x + 7 = 22$

   $\frac{3}{5}x = 15$

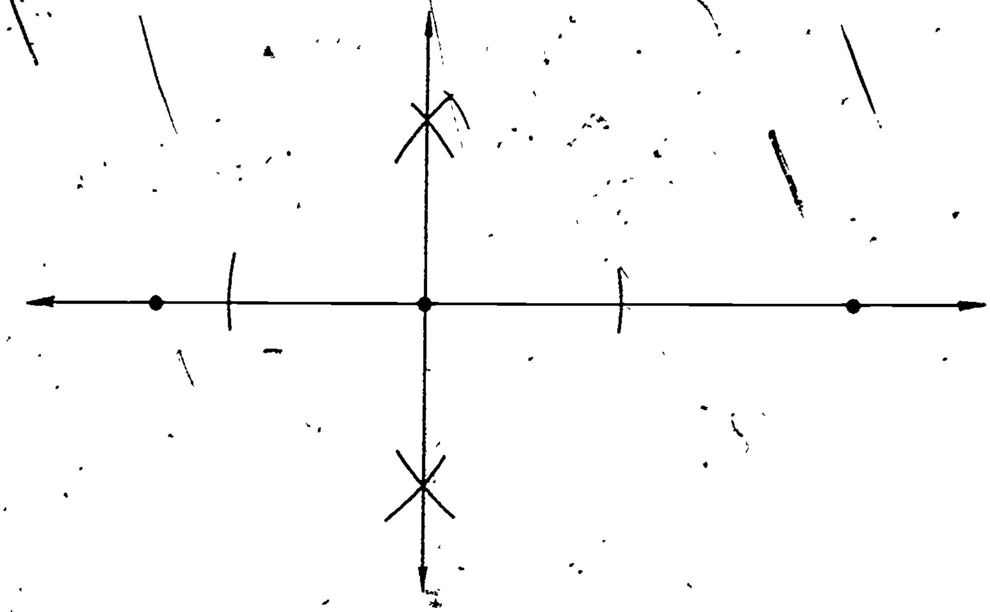
   $x = 25$

   (d) $\frac{4}{3}x + \frac{2}{5} = \frac{3}{5}$

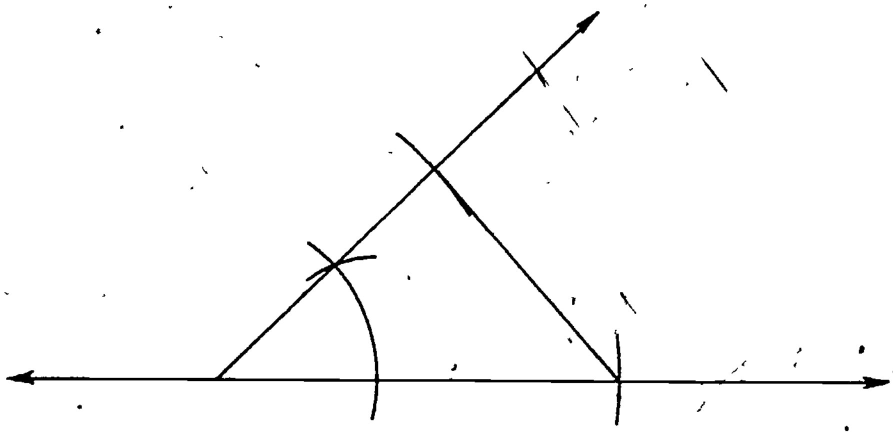
   $\frac{4}{3}x = 1$

   $x = \frac{3}{4}$

5.



6.



Chapter 11

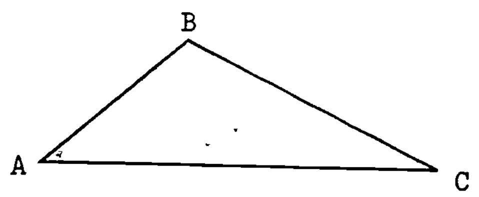
PARALLELISM

Chapter 11

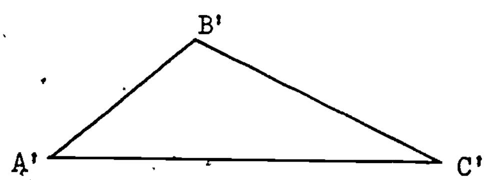
PARALLELISM

Congruence (Revisited)

In Chapter 9 you learned that two figures are congruent if a copy of one can be fitted exactly on the other. For example, if you make a tracing of $\triangle ABC$



and it fits exactly on $\triangle A'B'C'$



then we say that $\triangle ABC$ is congruent to $\triangle A'B'C'$. In symbols we would write

$$\triangle ABC \cong \triangle A'B'C'$$

where the symbol "$\cong$" is read "is congruent to".

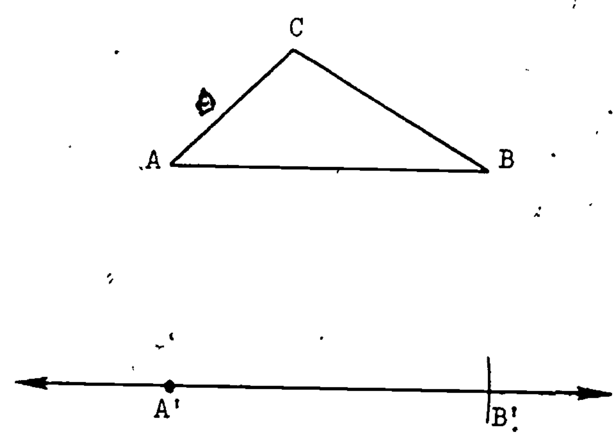
Let us now review the methods we used to copy a triangle using only a compass and ruler.

Class Discussion

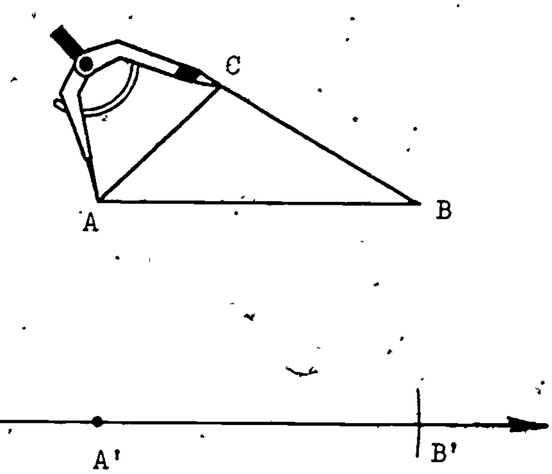
Copying a Triangle by SSS

On Page 11-1d you will find a triangle to be copied. Part of the line in the middle of the page will be used as one of the sides of the triangle. Follow the directions and figures below to help you copy the triangle.

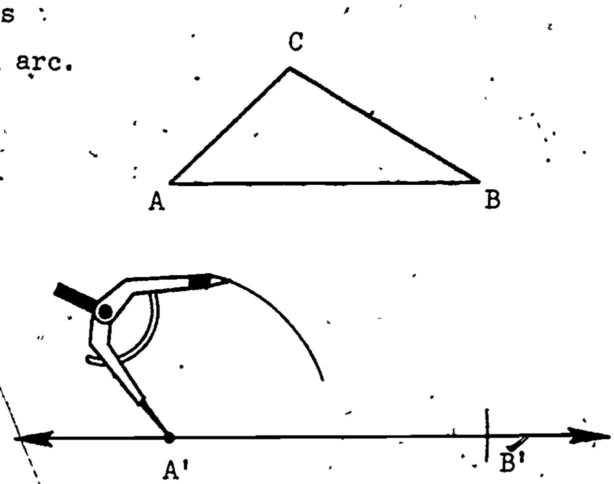
(a) Copy $\overline{AB}$ on to the line so that $\overline{AB}$ is congruent to $\overline{A'B'}$.



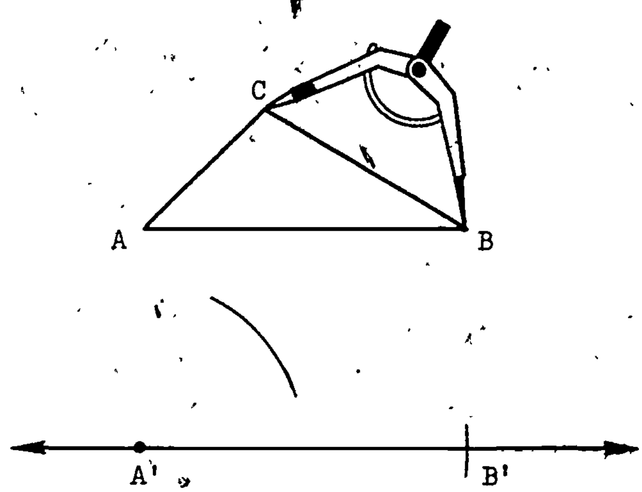
(b) Place the point of the compass on the vertex A and set your compass so that the pencil point falls on the vertex C.



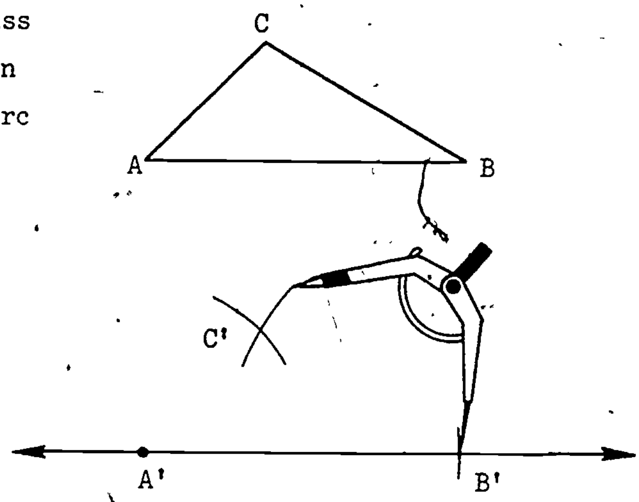
(c) Place the point of the compass on the point A' and draw an arc.



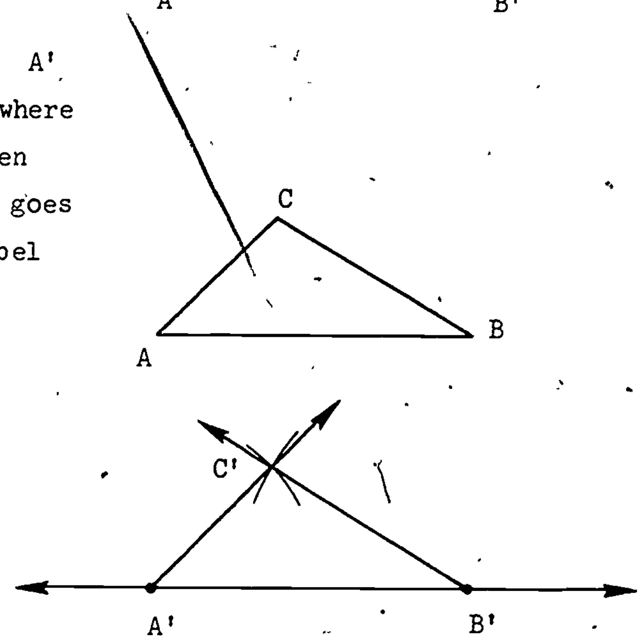
(d) Place the point of the compass on the vertex B and set your compass so that the pencil point falls on the vertex C.



(e) Place the point of the compass on the point B' and draw an arc that crosses the first arc you drew.



(f) Draw the ray that starts at A' and goes through the point where the two arcs intersect. Then draw the ray from B' that goes through the same point. Label this point C'.

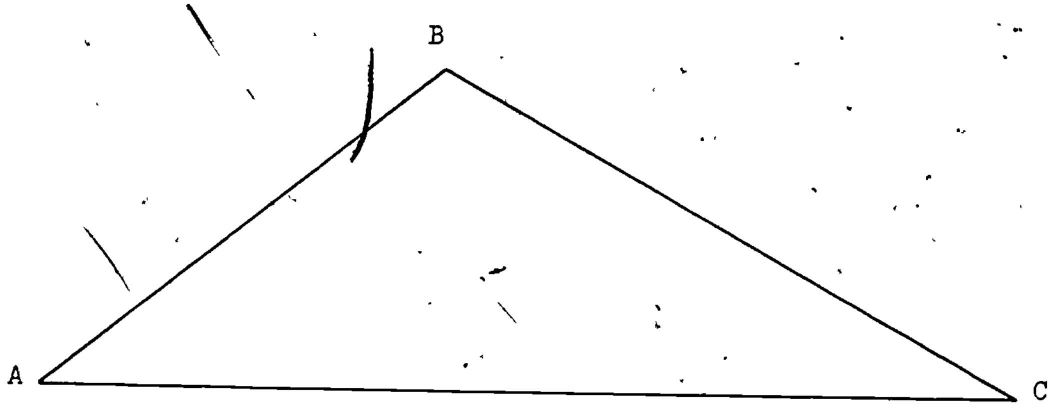


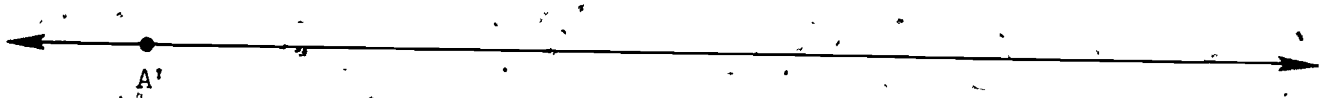
If you have been careful, $\triangle A'B'C' \cong \triangle ABC$.

> If three sides of one triangle are congruent to the sides of another triangle, then the two triangles are <u>congruent</u>.

This is called the SSS (Side, Side, Side) congruence property.

<u>Work Sheet</u>

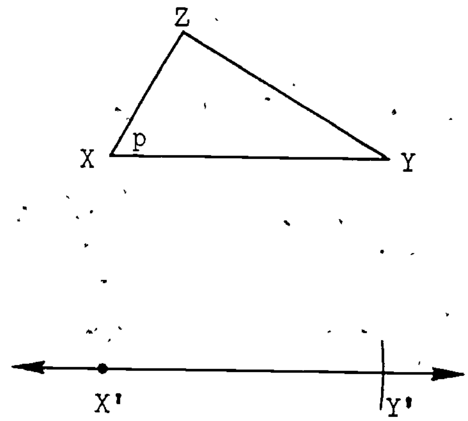




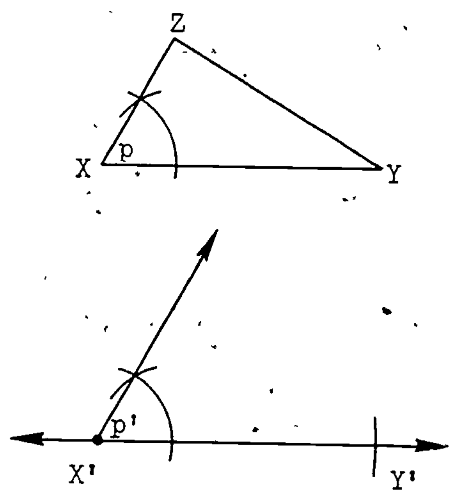
Copying a Triangle by SAS

On Page 11-1g you will find a triangle to be copied. Part of the line on the page is to be used as one side of the copied triangle. Follow the directions and figures below to help you copy the triangle.

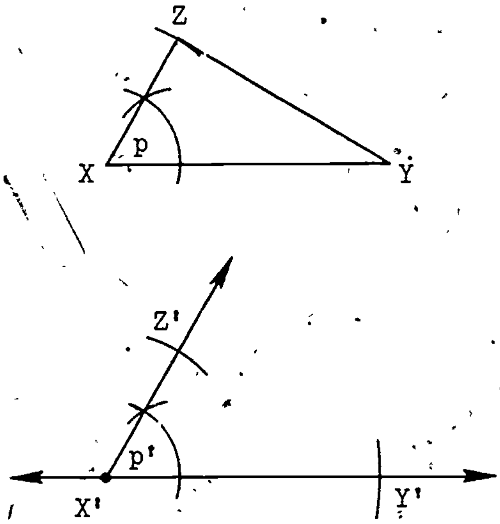
(a) Copy $\overline{XY}$ on to the line so that $\overline{XY}$ is congruent to $\overline{X'Y'}$.



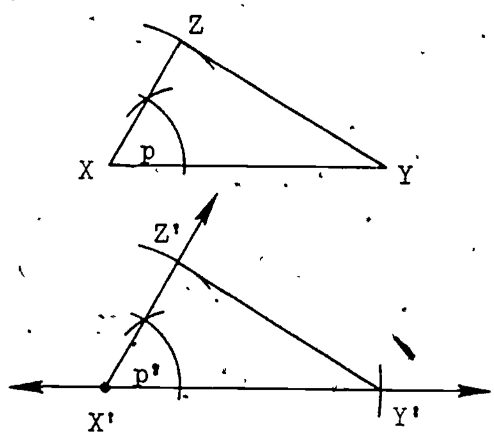
(b) Copy $\angle p$ at vertex X'.



(c) Copy $\overline{XZ}$ onto the ray of the angle you just copied.



(d) Draw $\overline{Z'Y'}$.



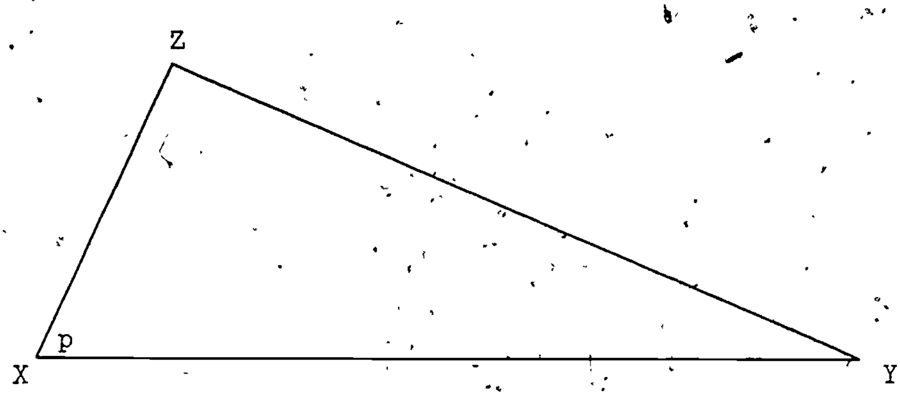
If you have been careful, $\triangle XYZ \cong \triangle X'Y'Z'$.

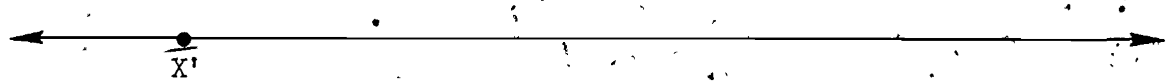
You can see that although a triangle has three sides and three angles we needed only three of these "parts", in this case two sides and an angle, to copy the triangle.

> If two sides and the angle they form of one triangle are congruent to two sides and the angle they form of another triangle, then the two triangles are congruent.

This is called the SAS (Side, Angle, Side) congruence property.

Work Sheet

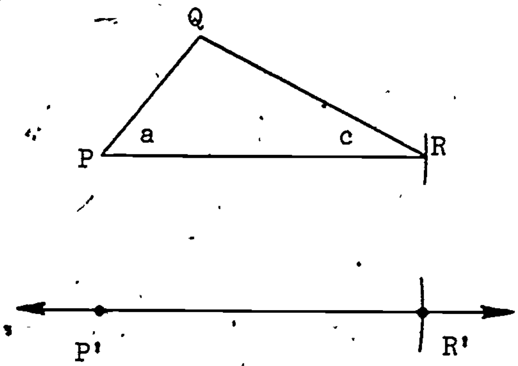




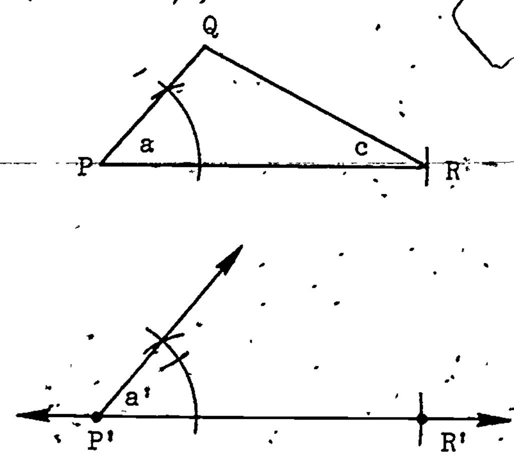
Copying a Triangle by ASA

On Page 11-1j you will find a triangle to be copied. Part of the line in the middle of the page is to be used as one side of the triangle to be copied. Follow the directions and figures below to help you copy the triangle.

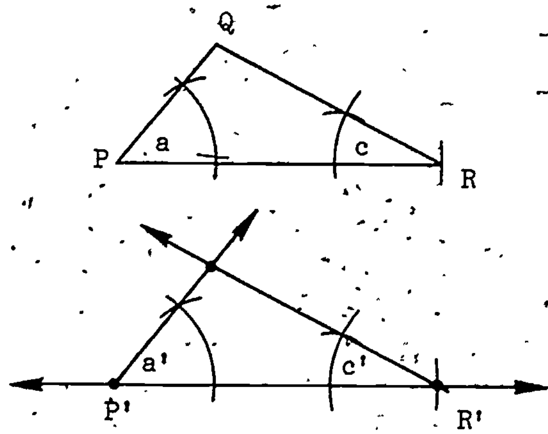
(a) Copy $\overline{PR}$ on to the line so that $\overline{PR}$ is congruent to $\overline{P'R'}$.



(b) Copy $\angle a$ at vertex P'.



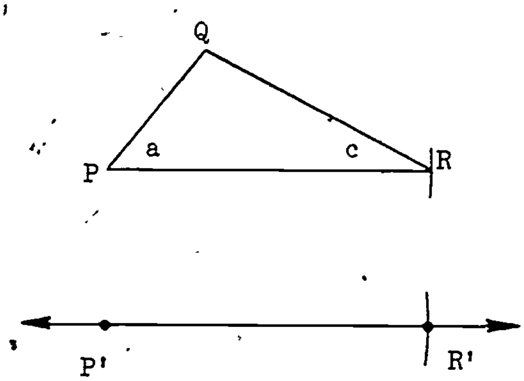
(c) Copy $\angle c$ at vertex R'.



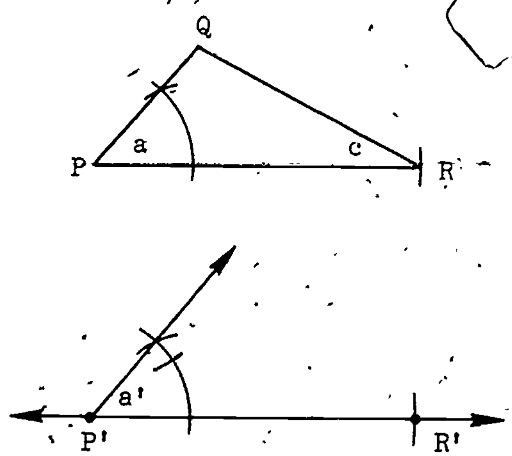
Copying a Triangle by ASA

On Page 11-1j you will find a triangle to be copied. Part of the line in the middle of the page is to be used as one side of the triangle to be copied. Follow the directions and figures below to help you copy the triangle.

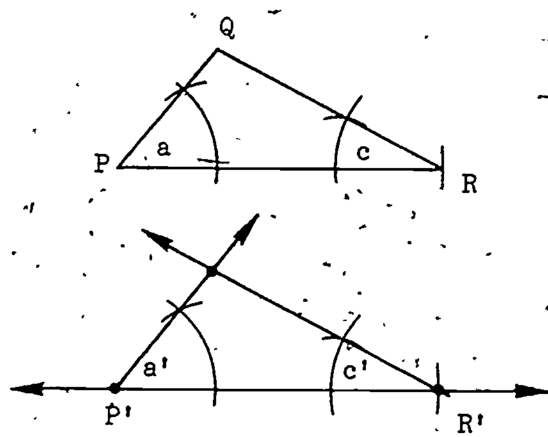
(a) Copy $\overline{PR}$ on to the line so that $\overline{PR}$ is congruent to $\overline{P'R'}$.



(b) Copy $\angle a$ at vertex P'.



(c) Copy $\angle c$ at vertex R'.



(d) Label the point Q'.

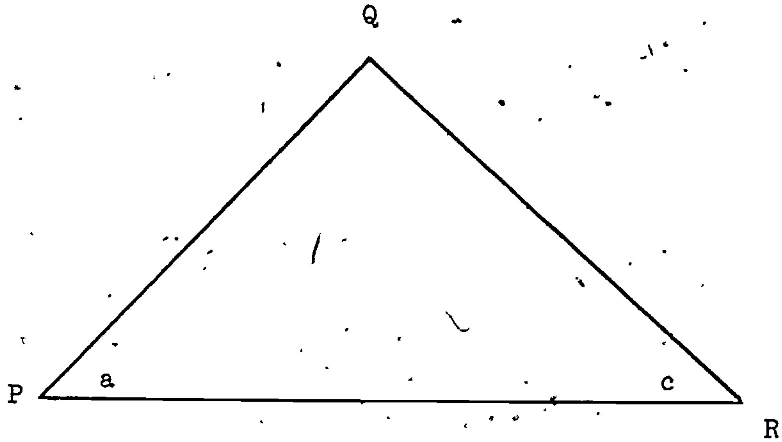
If you were careful, $\triangle PQR \cong \triangle P'Q'R'$.

You can see that all we needed to copy the triangle was two angles and a common side.

> If two angles and one side of one triangle are congruent to the corresponding two angles and one side of a second triangle, then the triangles are congruent.

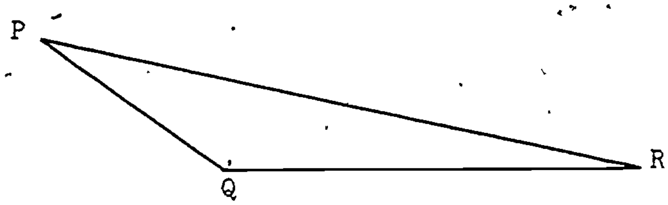
This is called the ASA (Angle, Side, Angle) congruence property.

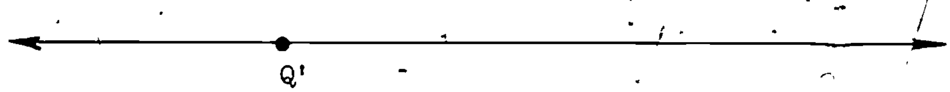
Work Sheet



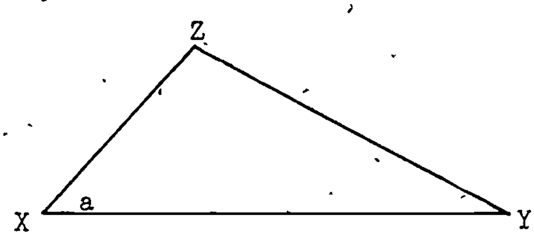
Exercises

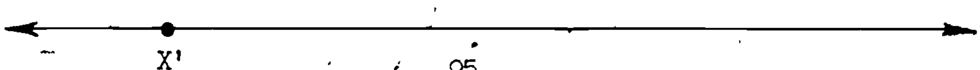
1. Using the SSS construction, copy the triangle below. Be sure to label the vertices. (Vertices is the plural of vertex.)



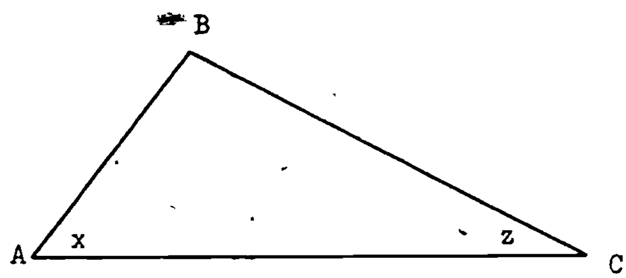


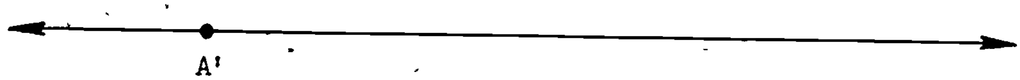
2. Using the SAS construction, copy the triangle below. Be sure to label the vertices.





3. Using the ASA construction, copy the triangle below. Be sure to label the vertices.





The Rhombus (Revisited)

Recall that a rhombus is a four-sided figure with all sides being congruent to each other. Also,

the diagonals of a rhombus:

(a) are perpendicular to each other,

(b) bisect each other,

and (c) bisect the angles at the end points of the diagonals.

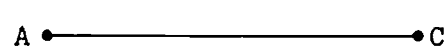
The class activity that follows will let you review how to use the rhombus to bisect a line segment and to bisect an angle.

Class Discussion

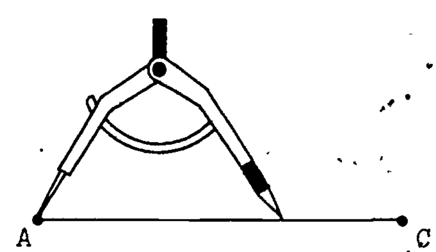
To bisect a line segment, all you need to do is to build a rhombus around it so that the line segment is one of the diagonals of the rhombus. The other diagonal will do the bisecting.

1. On Page 11-2c you will find a line segment to be bisected. Follow the directions and figures below to help you do the bisection:

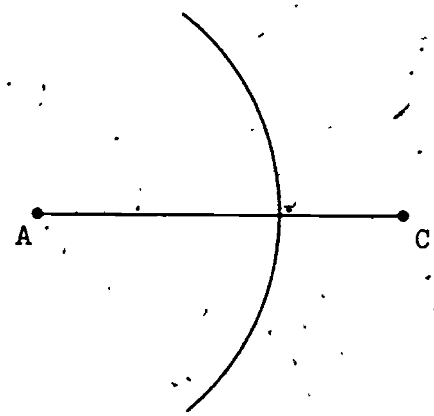
   (a) If $\overline{AC}$ is to be a diagonal then you must find a point B above the segment and a point D below the segment so that ABCD is a rhombus.



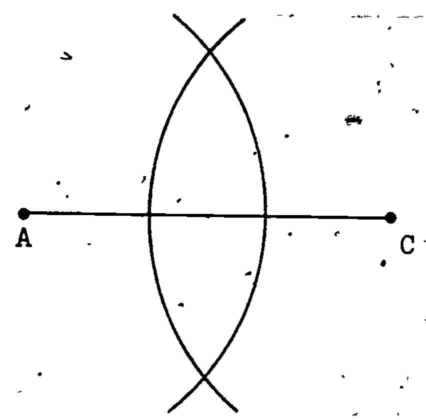
   (b) Set your compass so that the opening is wider than half the distance from A to C .



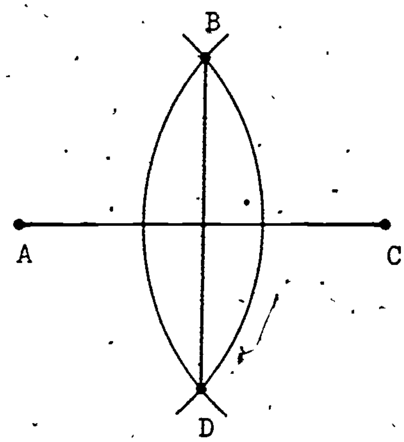
(c) Put the point of the compass on point A and draw an arc (about half a circle) that crosses $\overline{AC}$.



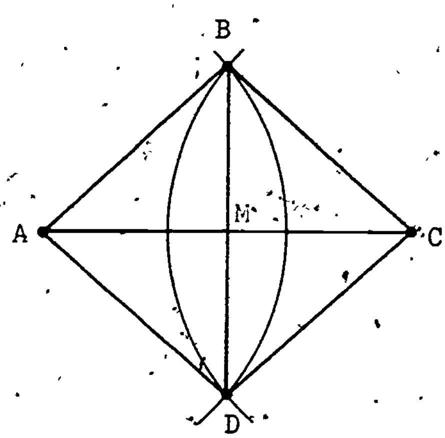
(d) Without changing your compass setting, put the point of the compass on point C and draw another arc so that it crosses the first arc you drew in two places.



(e). You now have located the point B above $\overline{AC}$ and point D below $\overline{AC}$. Draw $\overline{BD}$.

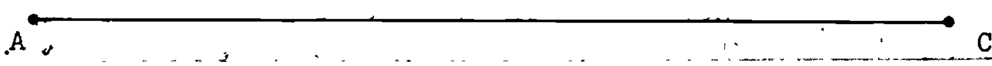


(f) Draw $\overline{AD}$, $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$. The figure ABCD is a rhombus. Label the intersection of $\overline{AC}$ and $\overline{BD}$ with the letter M.



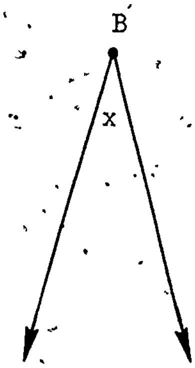
Because you know the diagonals of a rhombus bisect each other, you have, by "building a rhombus around $\overline{AC}$", bisected it at M.

Work Sheet

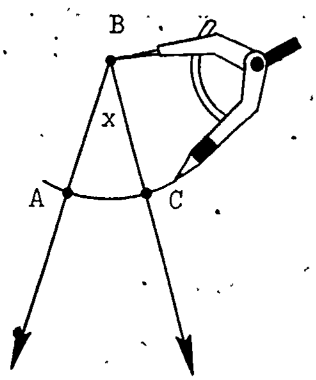


2. On Page 11-2f you will find an angle to be bisected. Follow the directions and figures below to help you do the bisection.

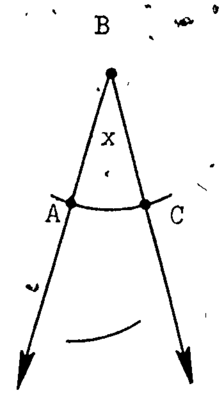
(a) To bisect $\angle x$ let it be one of the angles of a rhombus.



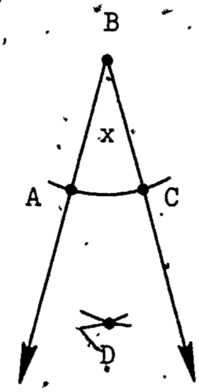
(b) Place the point of your compass on point B and draw an arc cutting both rays of the angle. This locates points A and C.



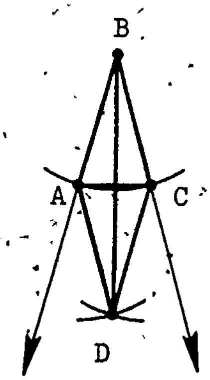
(c) Without changing your compass setting, place the point of the compass on point A and draw an arc.



(d) Without changing your compass setting, put the point of the compass on point C and draw an arc crossing the arc you have just drawn. You now have located point D .

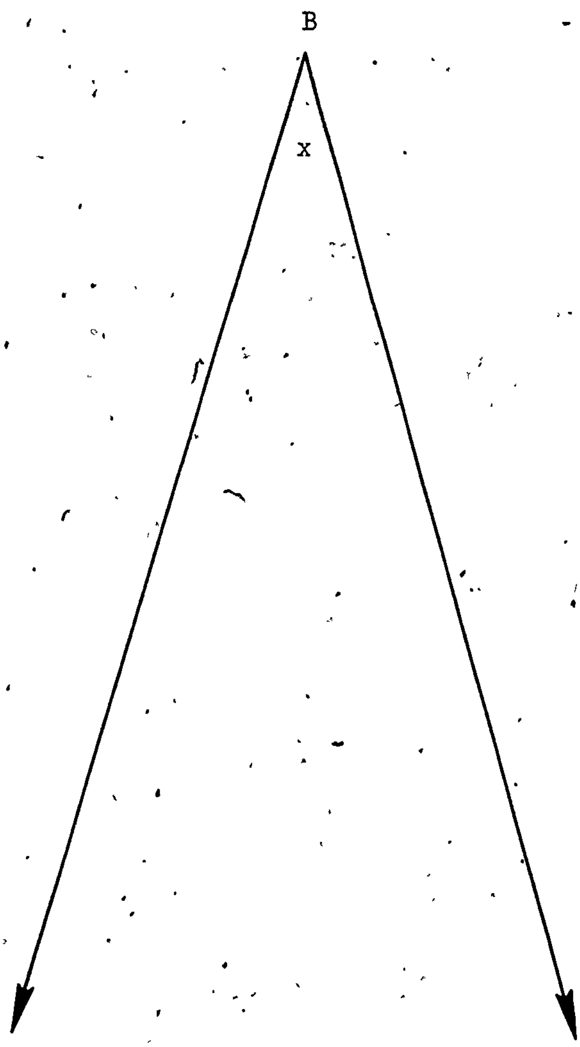


(e) Draw $\overline{AC}$, $\overline{BD}$, $\overline{AD}$, and $\overline{CD}$. The figure ABCD is a rhombus.



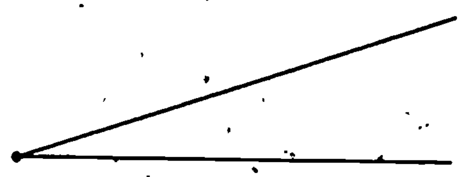
Because you know that the diagonals of a rhombus bisect the angles at their end points, you have, by "building a rhombus around $\angle x$", bisected it.

Work Sheet

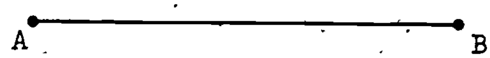


Exercises

1. Bisect the angle below.



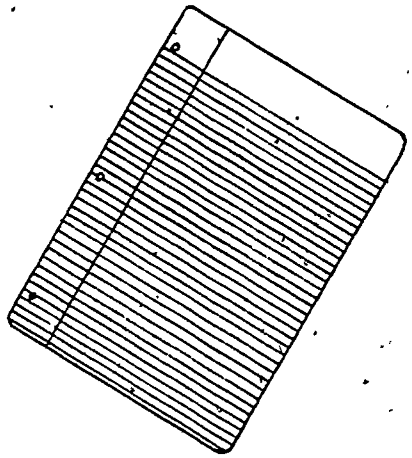
2. Bisect $\overline{AB}$.



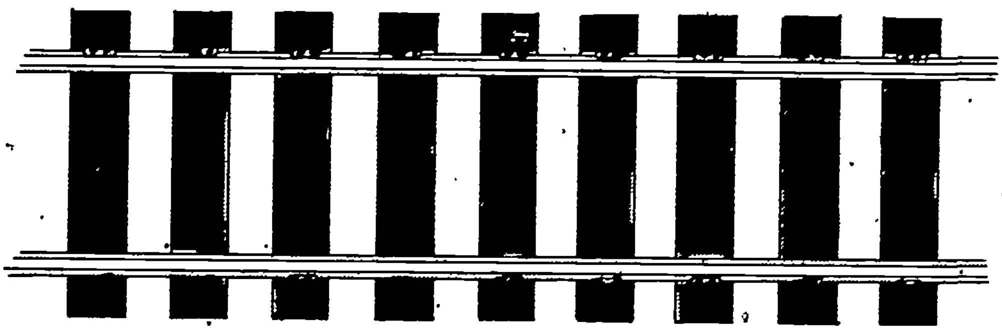
3. You know that the diagonals of a rhombus not only bisect each other but also are perpendicular to each other. Draw a line segment perpendicular to $\overline{PQ}$.

Parallel Lines in the Plane

You have seen many examples of parallel lines. For example, the lines on a ruled sheet of paper are parallel and they are all perpendicular to the edge of the paper.



The rails of a railroad track are parallel and they are both perpendicular to the ties.

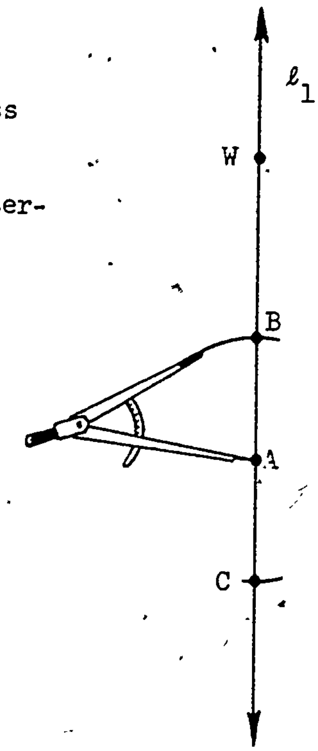


When we think of examples of parallel lines it almost always includes thinking about perpendicular lines. In the class activity that follows you will learn how to construct two parallel lines by making them perpendicular to a third line. As you might guess, we will use the rhombus to make this construction.

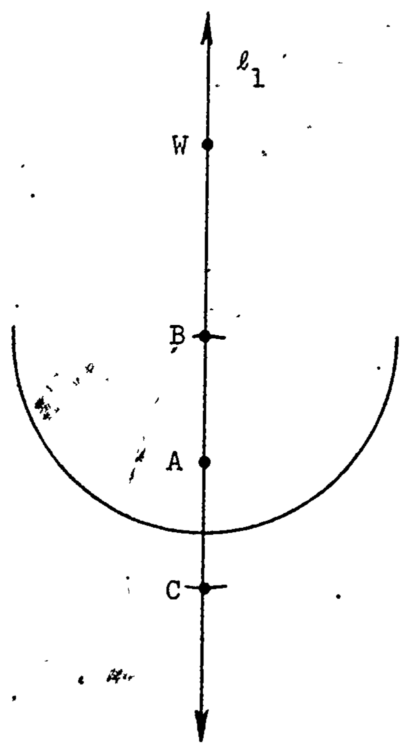
Class Discussion

On Page 11-3d a line has been drawn for you. Follow the directions below to construct two parallel lines that are perpendicular to the drawn line.

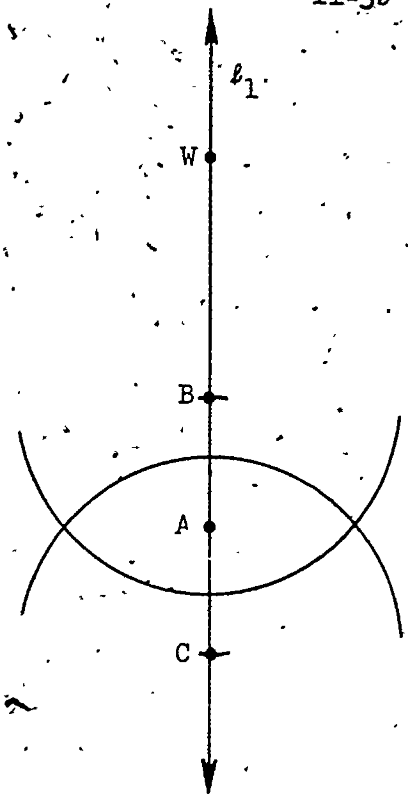
(a) Place the needle point of your compass on point A . Without changing your compass setting, draw an arc that intersects ℓ_1 above point A and an arc that intersects ℓ_1 below point A . Label the points of intersection B and C .



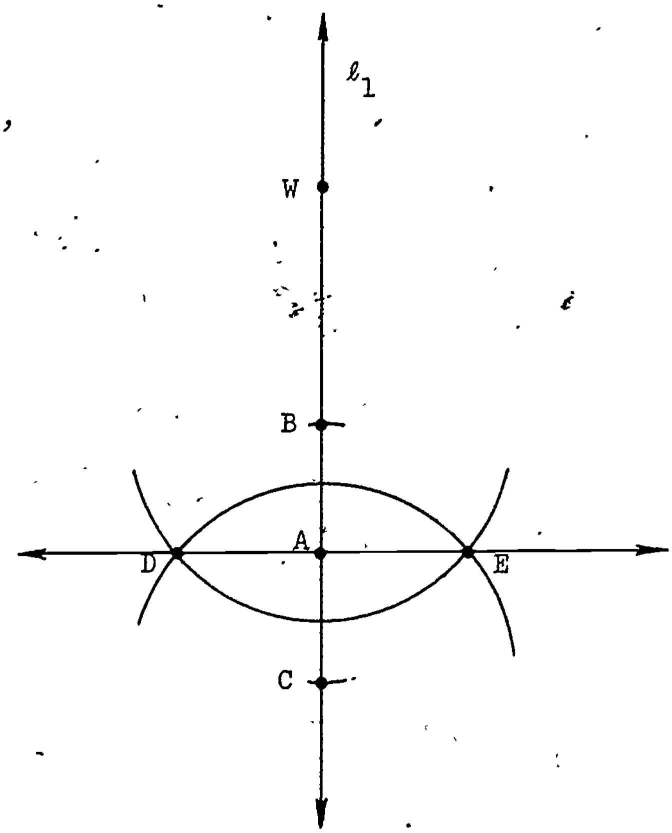
(b) Open your compass so that it is wider than the distance from A to B . Put the needle point on point B and draw an arc (about half a circle) that crosses ℓ_1 .



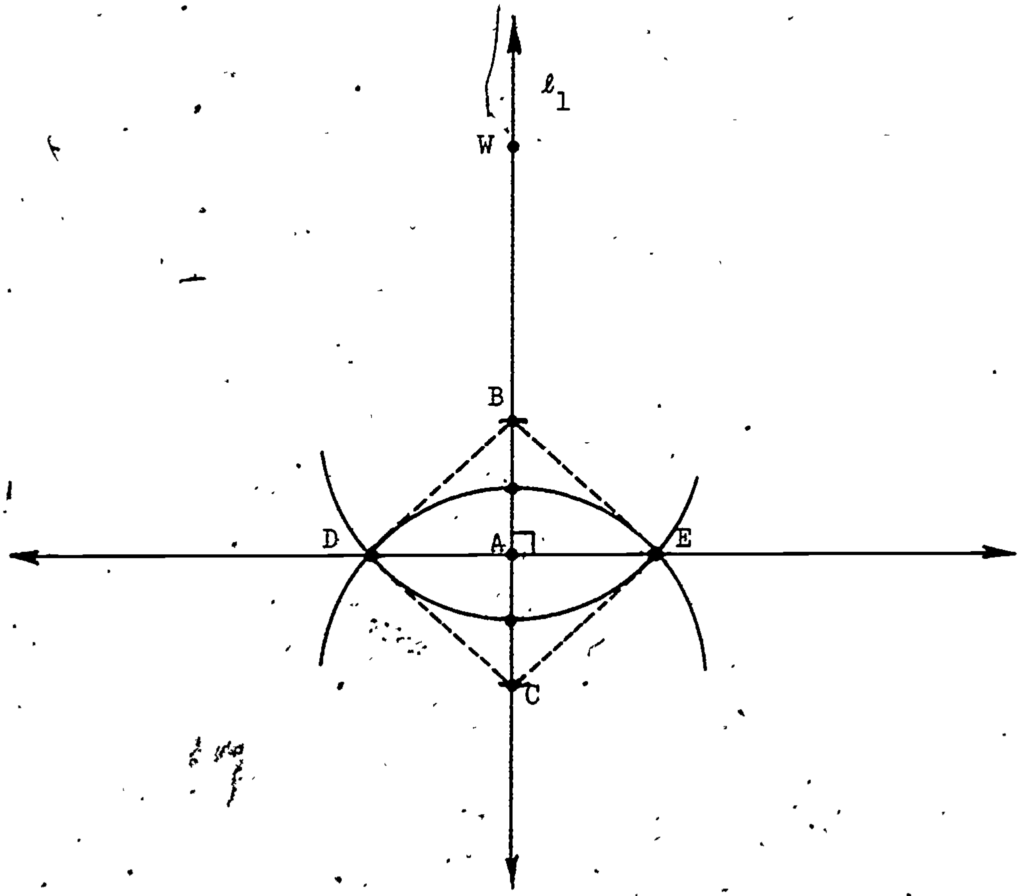
(c) <u>Without</u> changing your compass setting, put the needle point of the compass on point C and draw another arc that crosses the first arc you drew in two places.



(d) Label the points where the two arcs intersect D and E, and draw the <u>line</u> that passes through points D and E.



(e) Now let us look at the figure you just finished constructing. If we were to draw $\overline{BD}$, $\overline{DC}$, $\overline{CE}$, and $\overline{EB}$, the figure would be a rhombus.



We know that the diagonals of a rhombus are perpendicular to each other. We also know that the diagonal $\overline{DE}$ is part of the line that passes through points D and E. Therefore, <u>$\overline{DE}$ must be perpendicular to ℓ_1</u>. The little "$\square$" at the point where the two lines intersect is used to show that the two lines are perpendicular to each other.

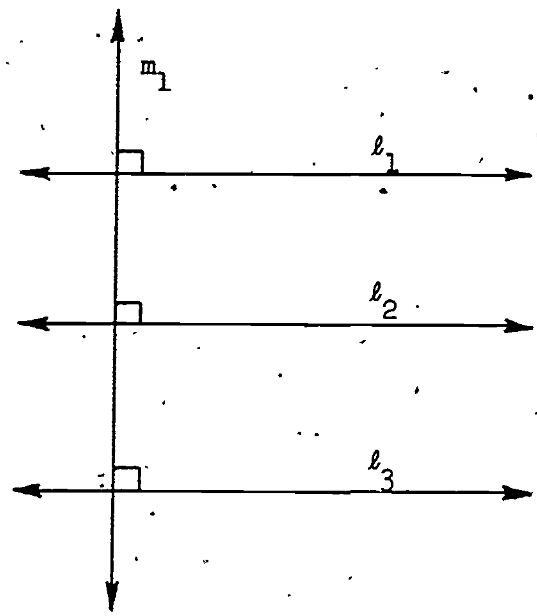
(f) Now construct a line through point W that is perpendicular to ℓ_1. Use the same method you just finished using.

When you have finished you will have constructed <u>two lines perpendicular to ℓ_1 and these two lines will be parallel.</u>

Work Sheet

Exercises

1. Pictured to the right are three lines perpendicular to a fourth line. In symbols we use " $\perp$ " to mean "is perpendicular to", and " $\parallel$ " to mean "is parallel to".



   (a) Is $\ell_1 \parallel \ell_2$? __________

   (b) Is $\ell_2 \parallel \ell_3$? __________

   (c) Is $\ell_1 \parallel \ell_3$? __________

2. Complete the sentences below. Use the drawing above to help you.

   (a) Two lines perpendicular to the same line are __________ to each other.

   (b) Two lines parallel to a third line are __________ to each other.

3. Suppose we have a line ℓ_1 and a point P not on ℓ_1.

   (a) How many lines do you think can go through P that are parallel to ℓ_1 ?

   ____

   (b) How many lines do you think can go through P that are perpendicular to ℓ_1 ? __________

BRAINBOOSTER.

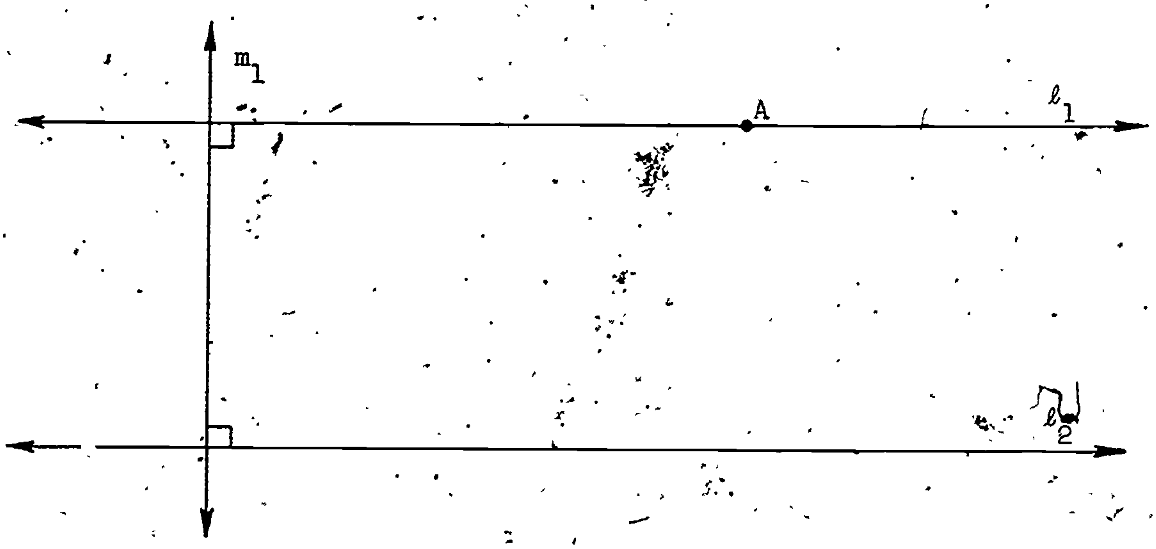
4. A line separates the plane into two regions. Into how many regions do <u>two</u> lines separate the plane:

   (a) if the lines are parallel? __________

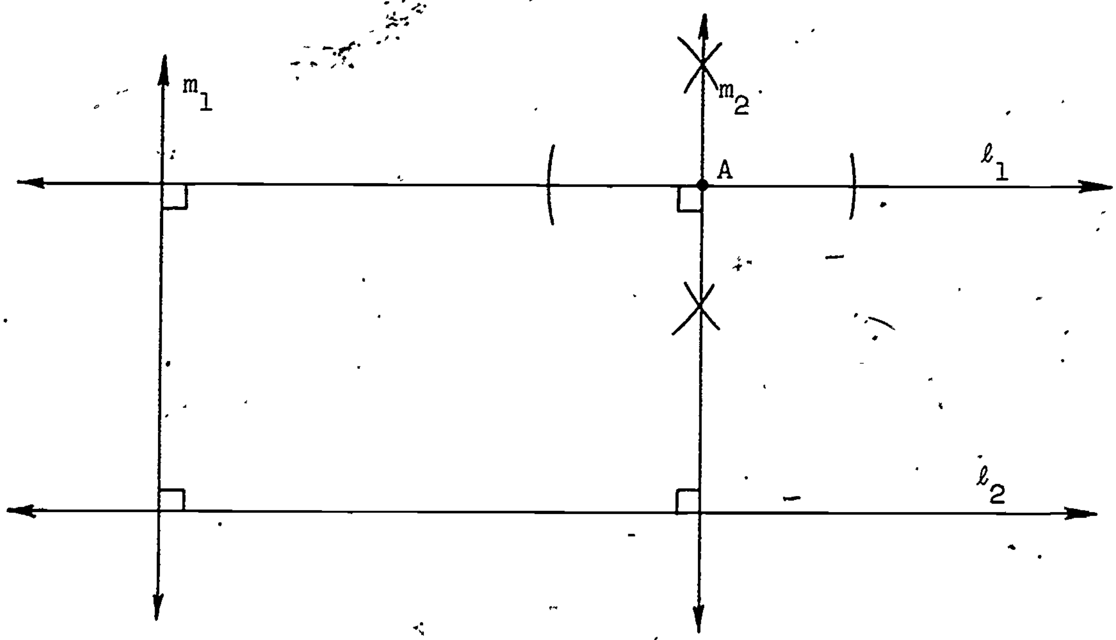
   (b) if the lines intersect? __________

Rectangles

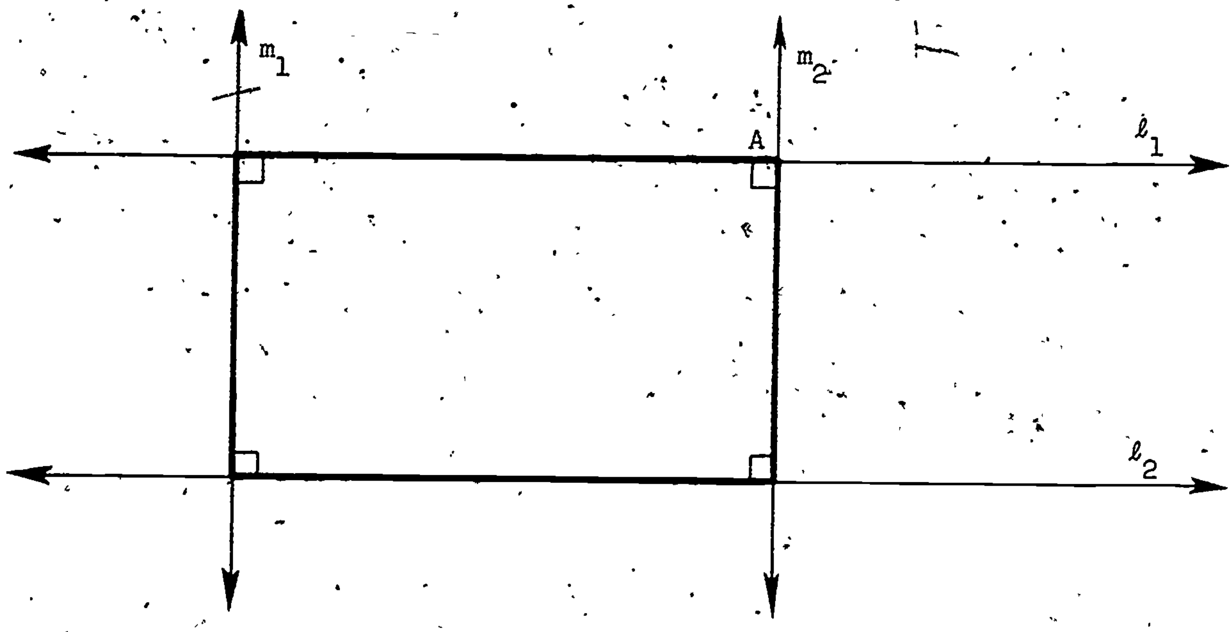
The drawing below is like the figure you constructed in the last class activity.



We pick a point on ℓ_1, say point A, and by building a rhombus we construct a line perpendicular to ℓ_1 at point A, and intersecting ℓ_2.



The figure outlined with the dark line is called a <u>rectangle</u>. Notice that m_1 and m_2 are perpendicular to ℓ_1; therefore m_1 and m_2 must be parallel to each other.



By looking at the way the above rectangle was constructed you see why we say the following things.

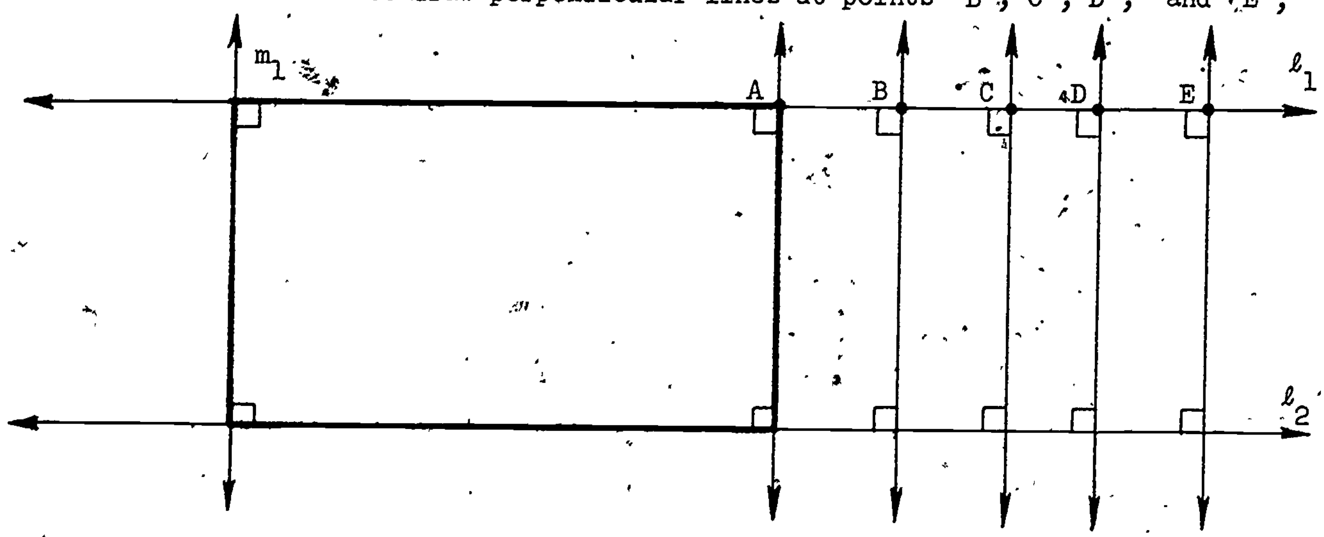
In a rectangle

(a) <u>the four angles are all right angles</u>,

and (b) <u>opposite sides are parallel</u>.

It is also true that <u>opposite sides are congruent</u>.

If we continue to draw perpendicular lines at points B, C, D, and E,



you can see that many rectangles have been formed. As opposite sides of a rectangle are congruent, then all of these segments are congruent to each other.

This fact now lets us say:

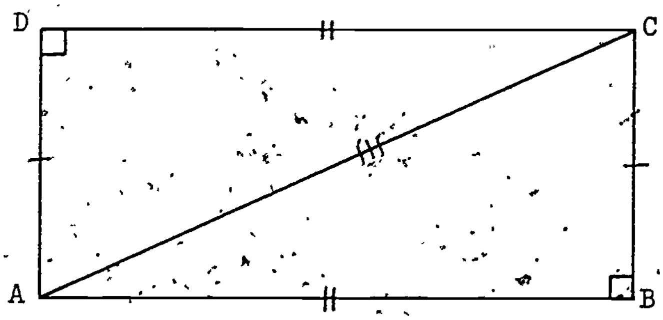
> the perpendicular distance between parallel lines is the same everywhere.

Class Discussion

Earlier in this chapter you learned that if

> three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent. (SSS property)

We will now use this idea to show that if you draw a diagonal of a rectangle, the two triangles formed are congruent.



We will agree that a line segment is congruent to itself.

1. Because figure ABCD is a rectangle, $\overline{AB} \cong$ ____________.
2. Because figure ABCD is a rectangle, $\overline{AD} \cong$ ____________.
3. Because a line segment is congruent to itself, $\overline{AC} \cong$ __________.
4. Then, by the __________ property of triangles,

$$\triangle ADC \cong \_\_\_\_\_\_\_\_$$

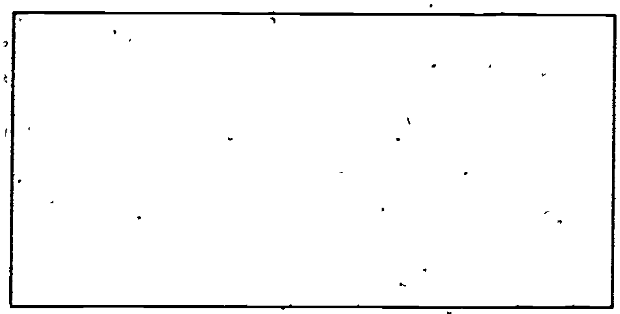
Exercises

1. (a) Construct two lines passing through points A and B that are perpendicular to m_1 .

(b) Now pick a point on one of the lines you drew perpendicular to m_1 and construct a line parallel to m_1 .

(c) What sort of figure has been formed? ________________

2. (a) Bisect each side of the rectangle below thus finding the midpoint of each side.

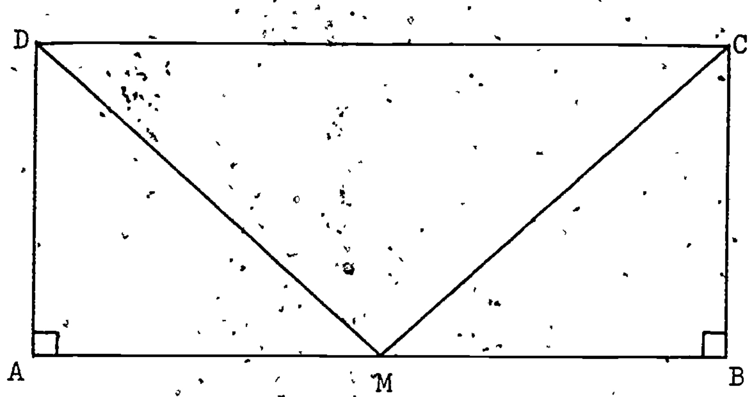


(b) Draw line segments from one midpoint to the next.

(c) What kind of figure is formed? ________________

BRAINBOOSTER.

3. In the rectangle below, point M is the midpoint of side $\overline{AB}$.

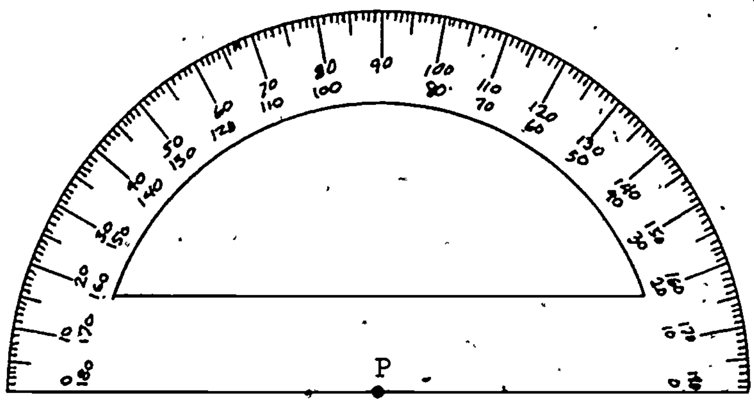


Try to give good reasons why $\triangle ADM \cong \triangle BCM$.

Hint: Use the SAS congruence property.

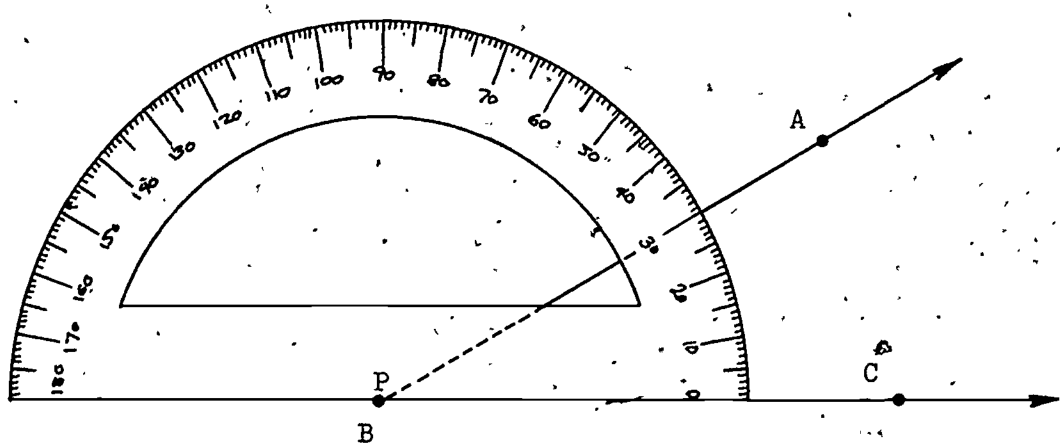
The Protractor

The instrument pictured below is called a protractor.



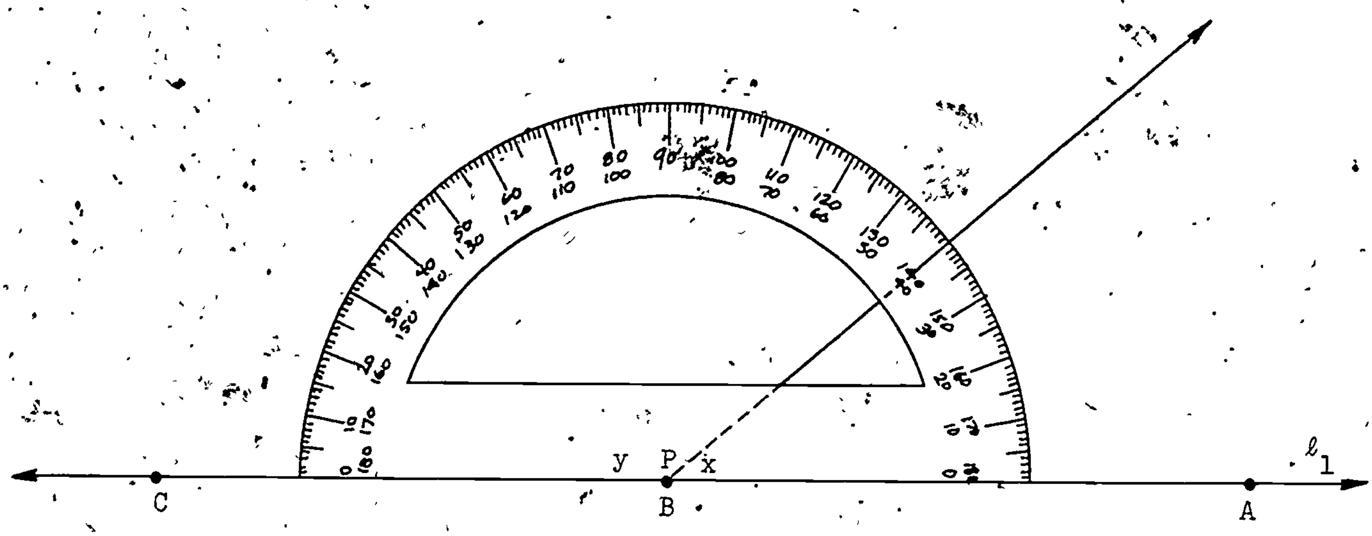
It is used to measure angles. Its semicircular scale (semicircular means "half a circle") is divided into 180 congruent angles.

To measure an angle you place the protractor on the angle so that point P on the protractor lies on the vertex of the angle and the edge of the protractor lies on one ray of the angle.



Now find the mark on the protractor that lies on the other ray of the angle. The number assigned to this mark is the measure, in degrees, of $\angle ABC$. In the picture above the measure of $\angle ABC$ (written $m\angle ABC$) is 30.

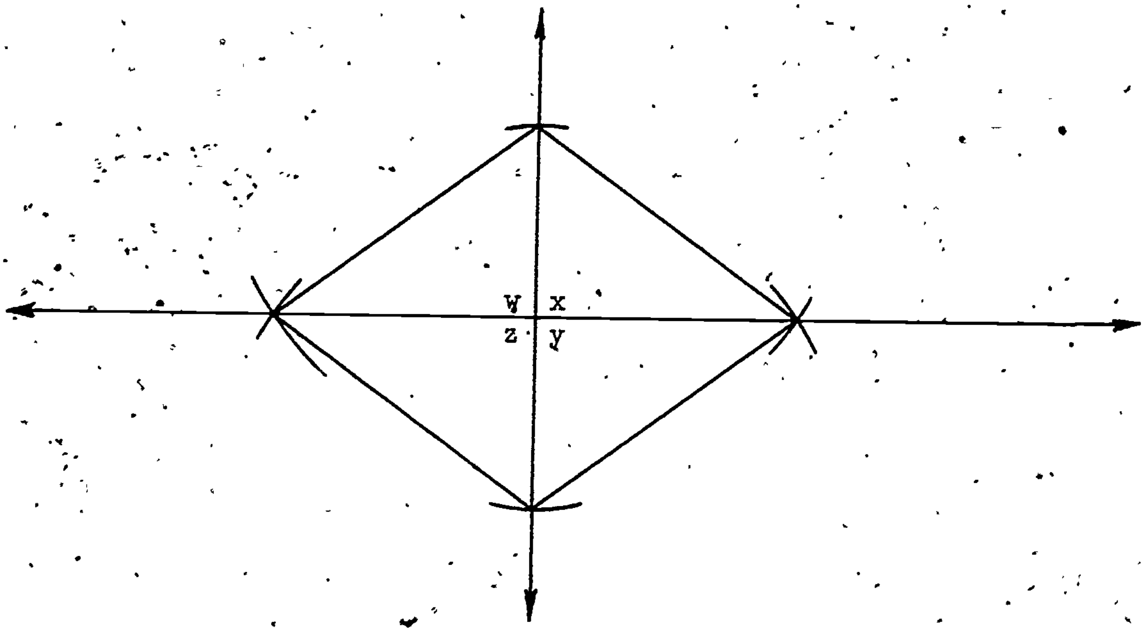
Class Discussion



Use the figure above to answer the following questions.

1. What is the measure of $\angle x$? ____________
2. What is the measure of $\angle y$? ____________
3. What is $m\angle x + m\angle y$? ____________
4. Notice that ℓ_1 can be thought of as being made up of two rays, $\overrightarrow{BC}$ and $\overrightarrow{BA}$. Because a line can be thought of in this way, mathematicians often call a line a straight angle. What is the measure of a straight angle? ____________

5. You know that the diagonals of a rhombus are perpendicular to each other. You also know that the four angles formed by the intersection of the diagonals are all congruent to each other.



(a) In the figure above, we know that $\angle w$, $\angle x$, $\angle y$, and $\angle z$ are all congruent. If the $m\angle x + m\angle w = 180$, what is the measure of $\angle w$? ________ of $\angle x$? ________

(b) What is the measure of $\angle z$? ________

(c) What is the measure of $\angle y$? ________

(d) $\angle w$, $\angle x$, $\angle y$, and $\angle z$ are all ________ (what kind?) angles.

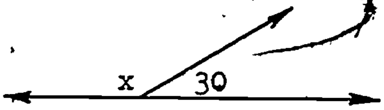
(e) Complete this sentence.

If two lines intersect and the four angles formed are all congruent to each other, then the two lines are ________ to each other.

Exercises

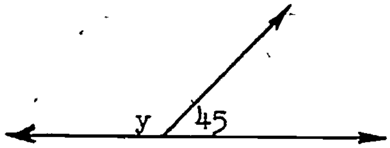
1. In the figures below, the measure of one angle is given to you. Find the measure of the other angle.

(a)



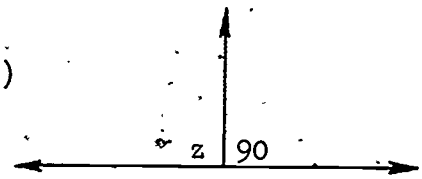
m ∠ x = ________

(b)



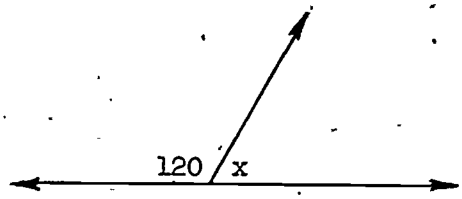
m ∠ y = ________

(c)



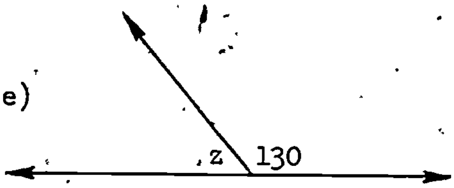
m ∠ z = ________

(d)



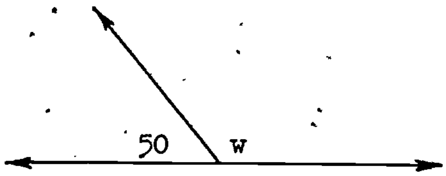
m ∠ x = ________

(e)



m ∠ z = ________

(f)

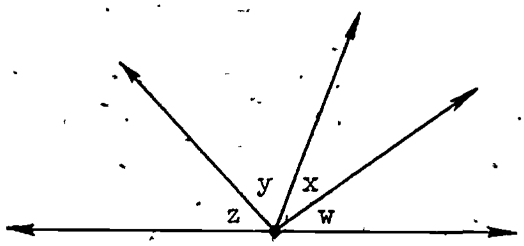


m ∠ w = ________

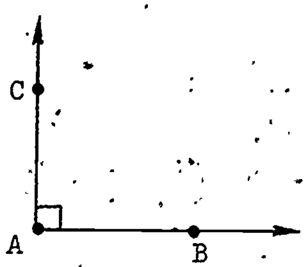
2. In the figure to the right, what is the sum

m ∠ w + m ∠ x + m ∠ y + m ∠ z?

____



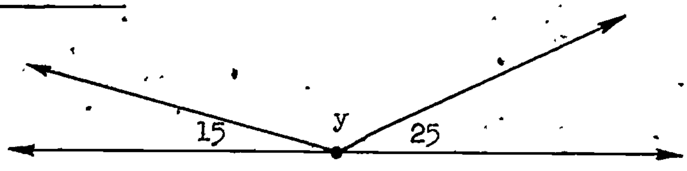
3. In the figure to the right, $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{AB}$. What is the measure of this angle? ________



BRAINBOOSTER.

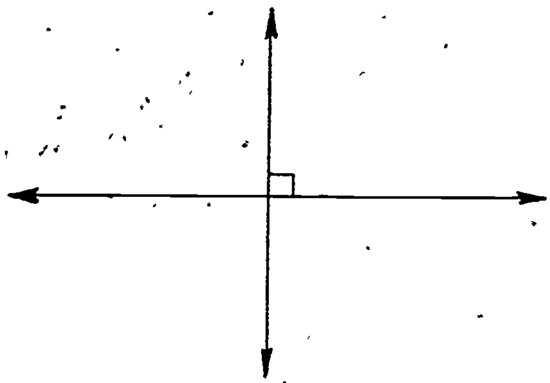
4. Find the measure of ∠ y .

m ∠ y = ________



Vertical Angles

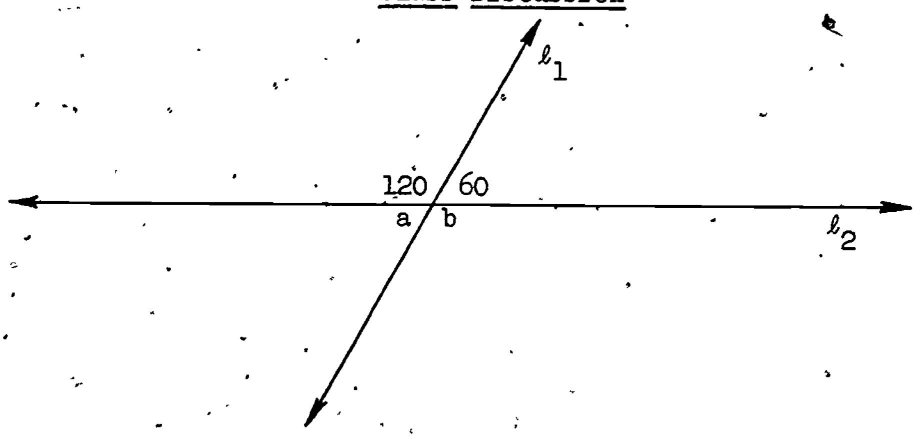
In the last lesson you found that if two lines intersect and the four angles formed are congruent to each other then the lines are perpendicular to each other.



The angles formed are called right angles and the measure of each is 90 .

Now let us look at two intersecting lines that are not perpendicular to each other.

Class Discussion



Use the above figure to help you answer the questions that follow.

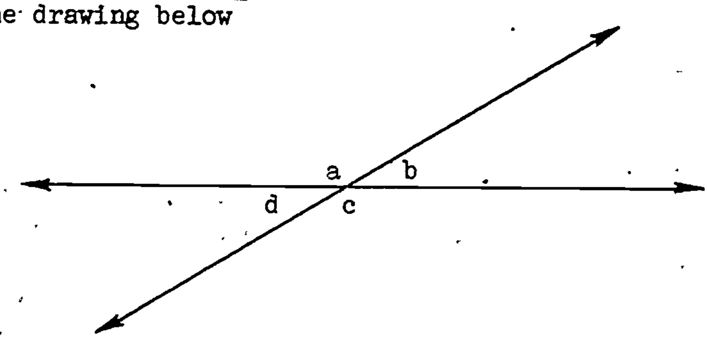
1. $120 + 60 =$ _________ .
2. $120 + m\angle a =$ _________ .
3. $m\angle a = 180 -$ (_________) .
4. $m\angle a =$ _________ .
5. $60 + m\angle b =$ _________ .
6. $m\angle b = 180 -$ (_________).
7. $m\angle b =$ _________ .

From these exercises we find that

$$m \angle a = 60 \quad \text{and} \quad m \angle b = 120 .$$

This shows us that when two lines intersect the opposite pairs of angles have the same measure. Angles formed in this way are called vertical angles.

In the drawing below



$\angle a$ and $\angle c$ are a pair of vertical angles and $\angle b$ and $\angle d$ are a pair of vertical angles.

We can show you that

any pair of vertical angles are congruent.

Keep in mind that when we talk about the measure of an angle, for example, $m \angle a$, we are talking about a number.

In the drawing above:

(a) $m \angle a + m \angle b = 180$ and $m \angle b + m \angle c = 180$.

(b) $m \angle a = 180 - m \angle b$ and $m \angle c = 180 - m \angle b$.

Now, since $180 - m \angle b$ is another name for both $m \angle a$ and $m \angle c$, then

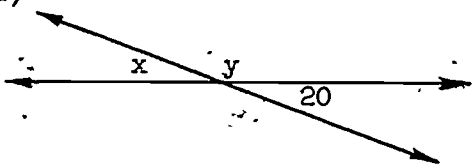
$$m \angle a = m \angle c .$$

By the same process we could show that $m \angle b = m \angle d$.

Exercises.

1. In each figure, find the measure of $\angle x$ and $\angle y$.

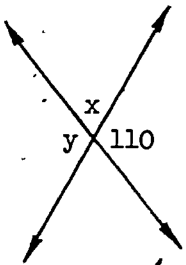
(a)



$m\angle x =$ ________.

$m\angle y =$ ________.

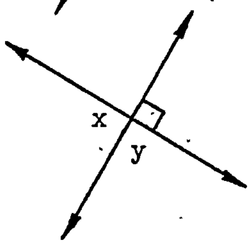
(b)



$m\angle x =$ ________.

$m\angle y =$ ________.

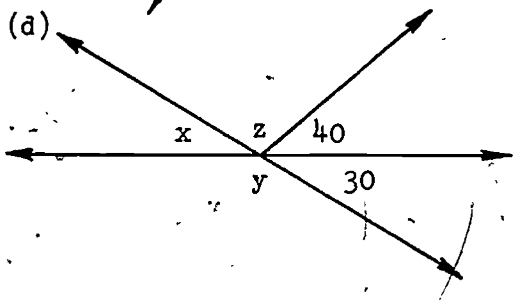
(c)



$m\angle x =$ ________.

$m\angle y =$ ________.

(d)



$m\angle x =$ ________.

$m\angle y =$ ________.

$m\angle z =$ ________.

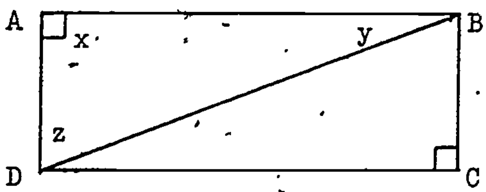
2. What is the measure of each angle formed by the intersection of the diagonals of a rhombus? ________

3. (a) The measure of each angle of a rectangle is ________.
   (b) What is the <u>sum</u> of the measures of the angles of a rectangle?

   ____

BRAINBOOSTER.

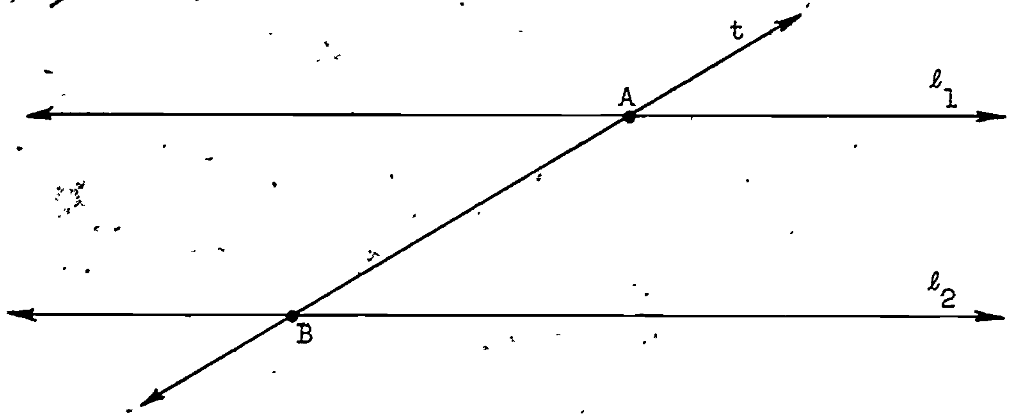
4. Pictured below is a rectangle with one diagonal drawn, thus forming two triangles.



Make a <u>guess</u> as to the sum: $m\angle x + m\angle y + m\angle z =$ ________.

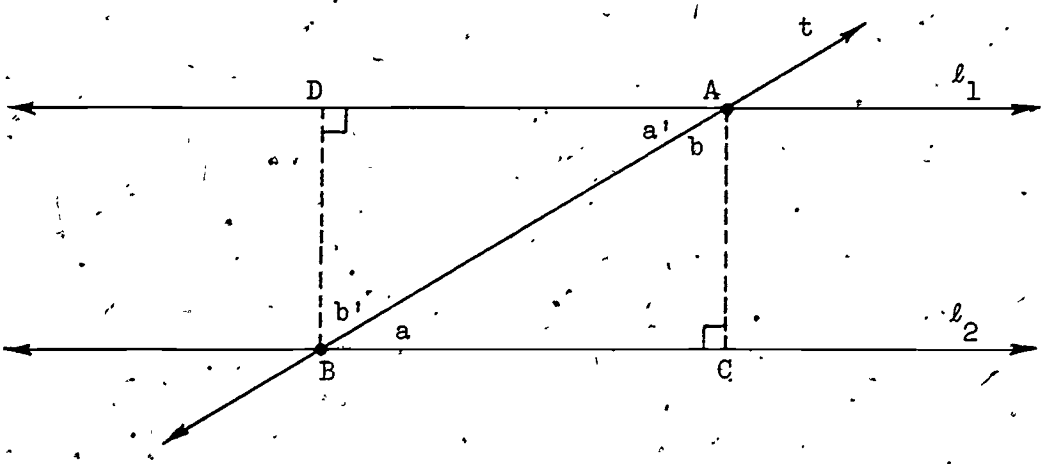
Transversals

Suppose that we have two parallel lines, ℓ_1 and ℓ_2, intersected by a third line, t.



The line t is called a transversal of ℓ_1 and ℓ_2.

Now, if we draw line segments at points A and B, perpendicular to ℓ_1 and ℓ_2, then the figure formed is a rectangle. Segment $\overline{AB}$ of the transversal now becomes the diagonal of the rectangle DACB.



In an earlier lesson you showed that if a diagonal of a rectangle is drawn then the two triangles formed are congruent. This tells us that in the drawing above

$$m\angle a = m\angle a'$$

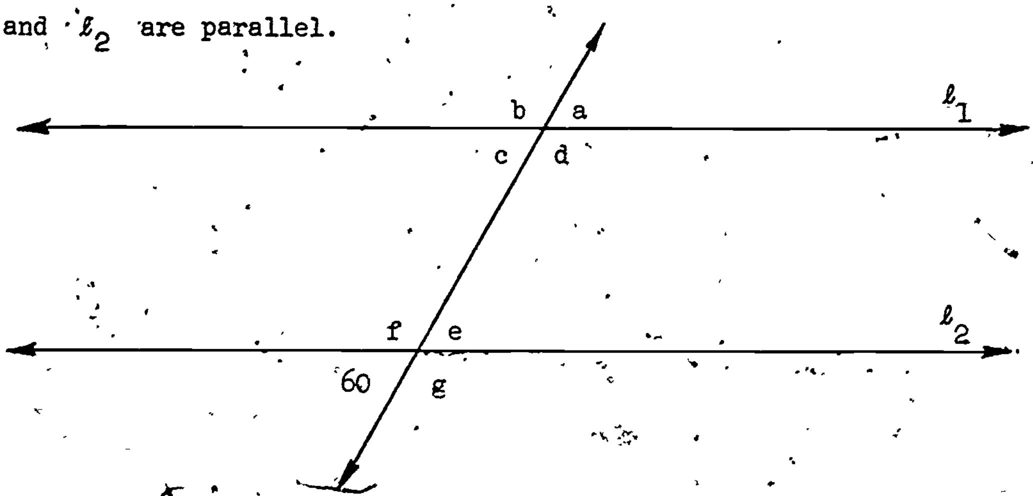
$$\text{and} \quad m\angle b = m\angle b'.$$

Pairs of angles located like $\angle a$ and $\angle a'$ or $\angle b$ and $\angle b'$ are called alternate interior angles, and they always have equal measures.

Class Discussion

Use the drawing below to answer the questions that follow.

ℓ_1 and ℓ_2 are parallel.



1. Because vertical angles have equal measure, $m\angle e$ = ________.

2. $\angle e$ and $\angle c$ are a pair of alternate interior angles. Therefore, $m\angle c$ = ________.

3. $m\angle g$ = ________.

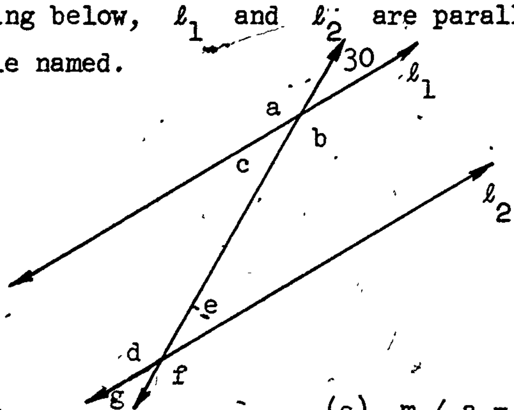
4. $m\angle f$ = ________.

5. $m\angle d$ = ________.

6. $m\angle b = m\angle d$. Why? ________________________

7. $m\angle f = m\angle d$. Why? ________________________

8. In the drawing below, ℓ_1 and ℓ_2 are parallel. Find the measure of each angle named.



(a) $m\angle c$ = ________ (e) $m\angle a$ = ________

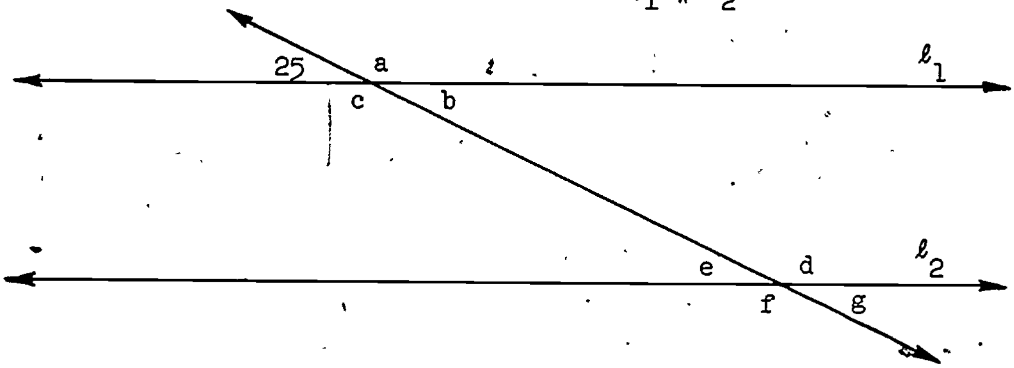
(b) $m\angle e$ = ________ (f) $m\angle b$ = ________

(c) $m\angle g$ = ________ (g) $m\angle d$ = ________

(d) $m\angle f$ = ________

Exercises

1. Find the measure of each angle named. $\ell_1 \parallel \ell_2$.



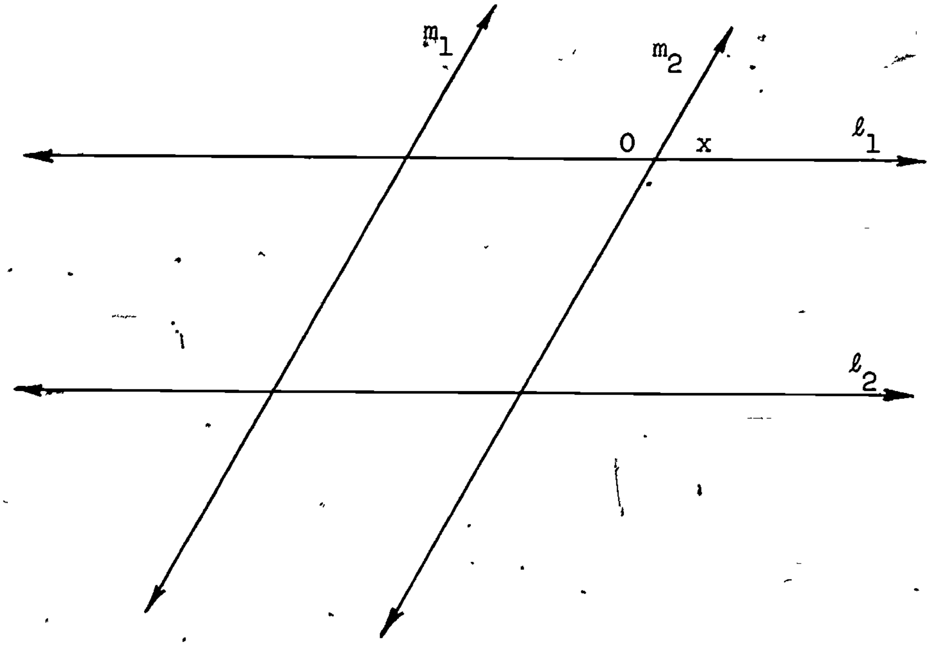
(a) $m\angle a =$ ________ (e) $m\angle e =$ ________

(b) $m\angle b =$ ________ (f) $m\angle f =$ ________

(c) $m\angle c =$ ________ (g) $m\angle g =$ ________

(d) $m\angle d =$ ________

2. In the figure below ℓ_1 and ℓ_2 are parallel and m_1 and m_2 are parallel. Mark with an "x" all angles congruent to $\angle x$ and with an "O" all angles congruent to $\angle O$.

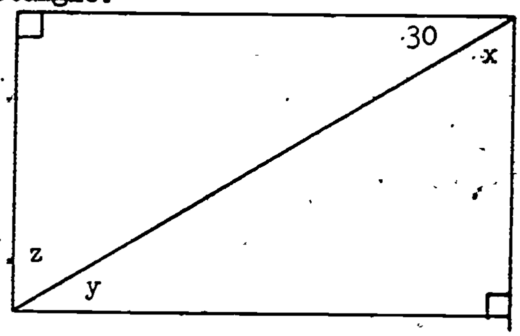


3. The figure to the right is a rectangle.

(a) m $\angle$ y = ________.

(b) m $\angle$ x = ________.

(c) m $\angle$ z = ________.

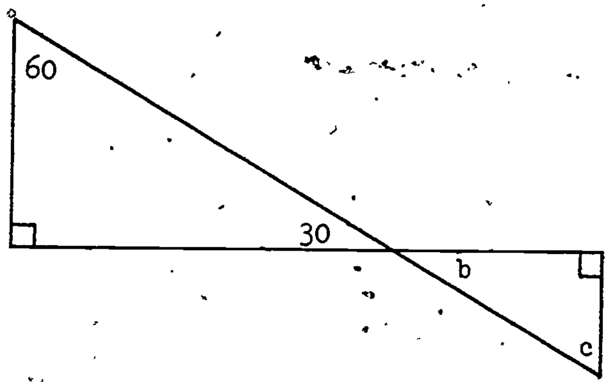


BRAINBOOSTER.

4. Find the measures of $\angle$ b and $\angle$ c .

(a) m $\angle$ b = ________.

(b) m $\angle$ c = ________.

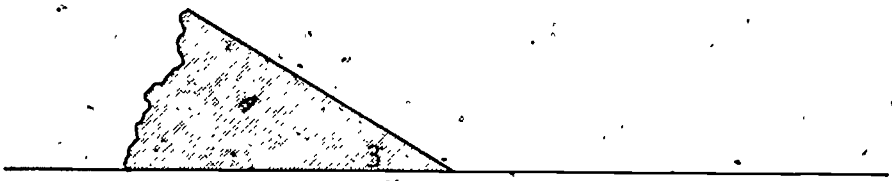


Triangles

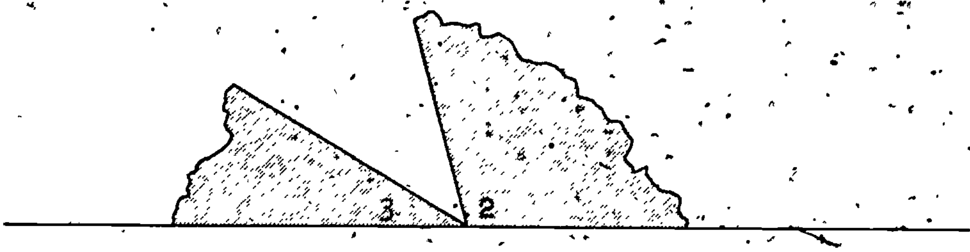
By now you have probably figured out that the sum of the measures of the angles of a triangle is 180°. We will perform an experiment that should convince you that this is true.

Class Discussion

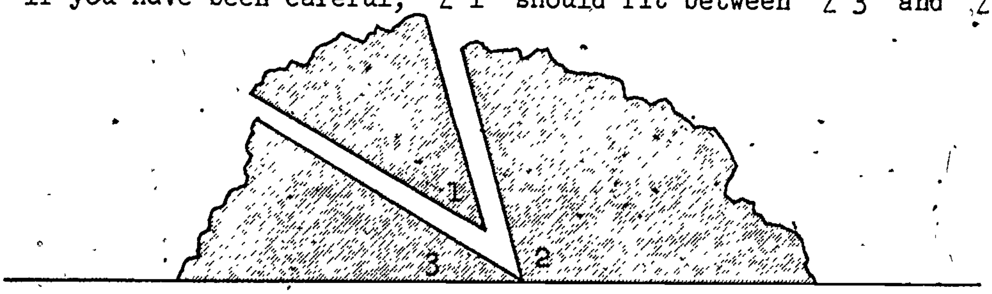
1. On Page 11-8d is a worksheet with a triangle drawn at the top of the page and a line drawn at the bottom. Take this sheet out of your notebook.
2. With a pair of scissors, carefully cut out the triangular region.
3. Now tear off (do not cut with your scissors) each corner of the triangle.
4. On the line drawn at the bottom of your work sheet, place $\angle 3$ so that one side lies on the line.



5. Now place $\angle 2$ as is shown below.



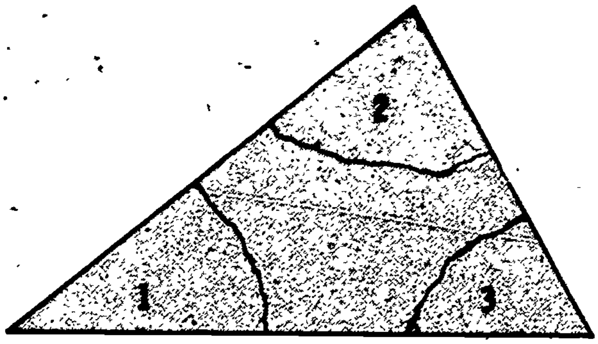
6. If you have been careful, $\angle 1$ should fit between $\angle 3$ and $\angle 2$.



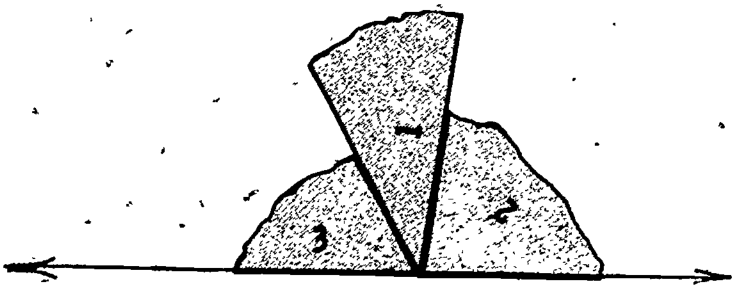
7. The sum of the measures of these three angles is the same as the measure of a straight angle. What is the measure of a straight angle? ____________

Let us summarize what we just did.

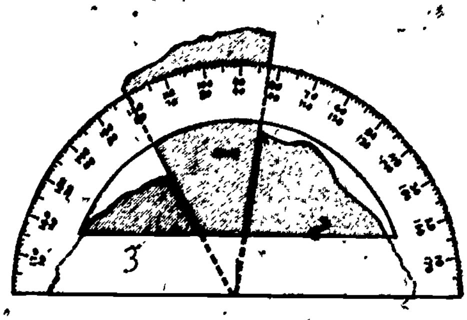
We took a cut-out triangle and tore off the corners.



Then we put these torn-off corners together.



When placed together the sum of the measures of the three angles is the same as the measure of a straight angle, which we know to be 180 .

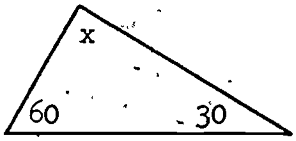


The sum of the measures of the angles of a triangle is 180 .

Exercises

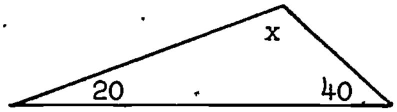
1. In each case, find the measure of $\angle x$.

(a)



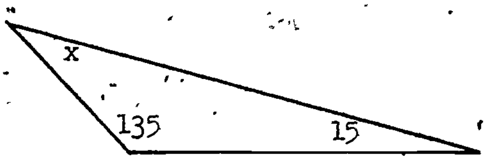
$m\angle x =$ ________

(b)



$m\angle x =$ ________

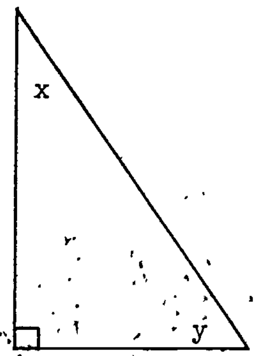
(c)



$m\angle x =$ ________

2. In the triangle to the right,

$m\angle x + m\angle y =$ ________.

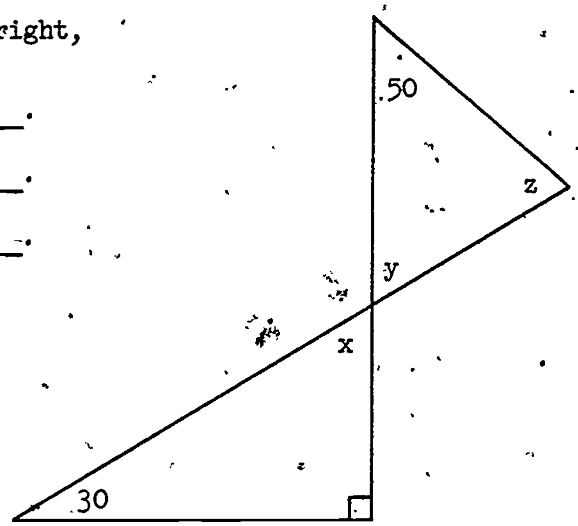


3. In the figure to the right,

(a) $m \angle x =$ ________.

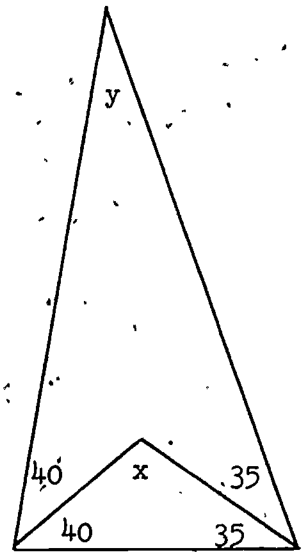
(b) $m \angle y =$ ________.

(c) $m \angle z =$ ________.



BRAINBOOSTER.

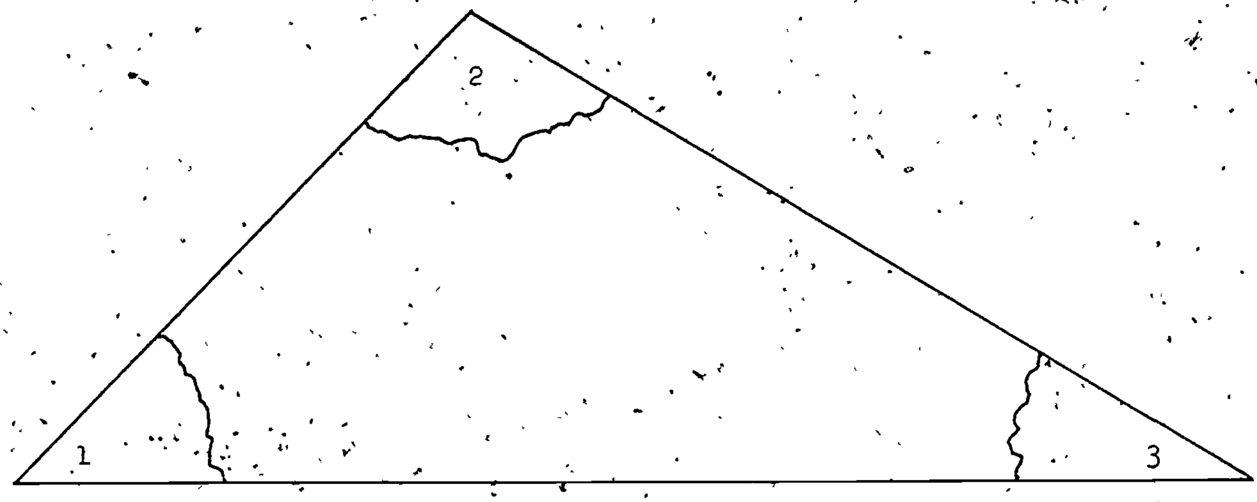
4.



(a) $m \angle x =$ ________

(b) $m \angle y =$ ________

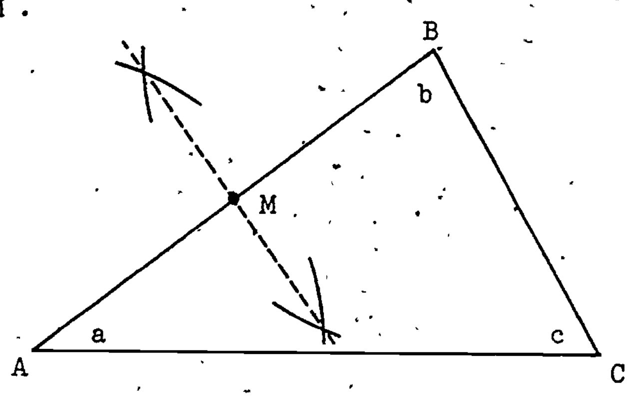
Work Sheet



Parallelograms

Class Discussion

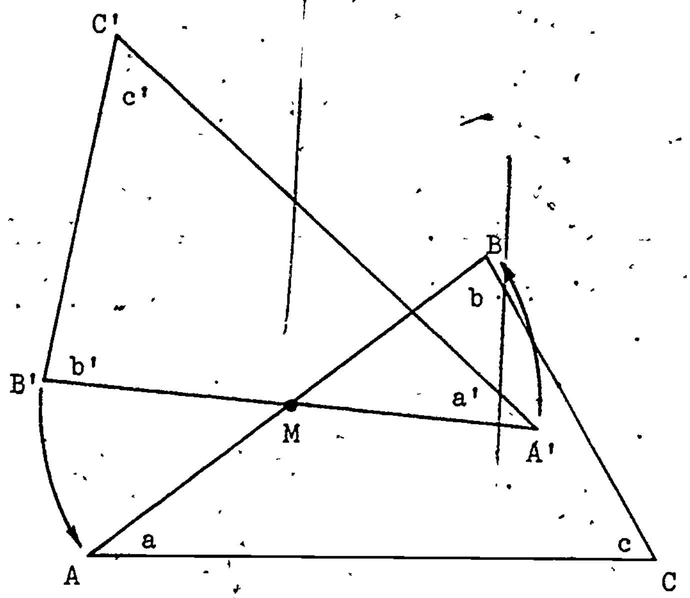
Pictured below is a triangle with side $\overline{AB}$ bisected so as to find the midpoint M .



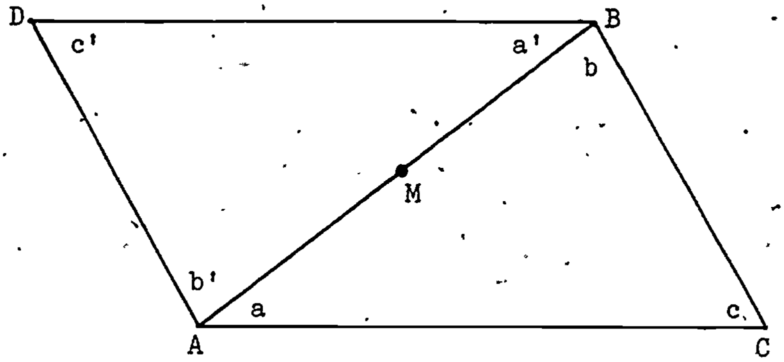
1. Take a piece of tracing paper and carefully make a tracing of $\triangle ABC$. Use a straightedge. Mark point M' on your tracing.

2. Without removing your tracing paper, stick the needle point of your compass through the tracing paper at point M' and into point M .

3. Now turn your tracing paper so that point B' falls on point A and point A' falls on point B .



4. We now have a new position for point C which we will call point D.



The figure formed by making this turn is called a <u>parallelogram</u>.

Class Discussion

1. In the figure above,

   (a) $m\angle a + m\angle b + m\angle c =$ ______________;

   (b) $m\angle a' + m\angle b' + m\angle c' =$ ______________.

   (c) From your answers to (a) and (b) what would you say was the sum of the measures of the angles of a parallelogram?

   ____

2. In the figure above, $\angle c$ is said to be <u>opposite</u> $\angle c'$ and $\angle DBC$ is said to be <u>opposite</u> $\angle CAD$.

   (a) Is it true that $m\angle c = m\angle c'$? ______________

   (b) Is it true that $m\angle a + m\angle b' = m\angle a' + m\angle b$? ______________

   (c) From your answers to (a) and (b) complete the following statement:

   The opposite angles of a parallelogram are ______________ in measure.

3. In the figure above,

   (a) Is it true that $\overline{AC} \cong \overline{BD}$? ______________

   (b) Is it true that $\overline{AD} \cong \overline{BC}$? ______________

   (c) Complete the following sentence:

   The opposite sides of a parallelogram are ______________ in measure.

To summarize:

In a parallelogram:

(a) opposite sides are equal in measure.

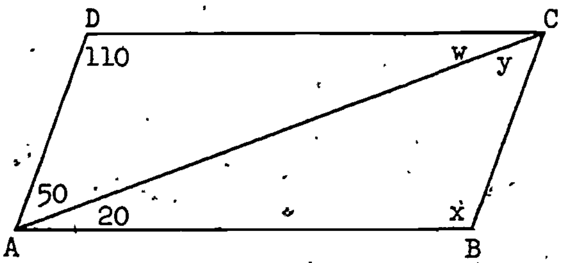
(b) opposite angles are equal in measure.

(c) the sum of the measures of the angles of a parallelogram is 360 .

(d) it is also true that (as you would suspect) the opposite sides of a parallelogram are parallel to each other.

Exercises

1. The figure to the right is a parallelogram.



   (a) What is the measure of $\angle x$? ________

   (b) What is the measure of $\angle w + \angle y$? ________

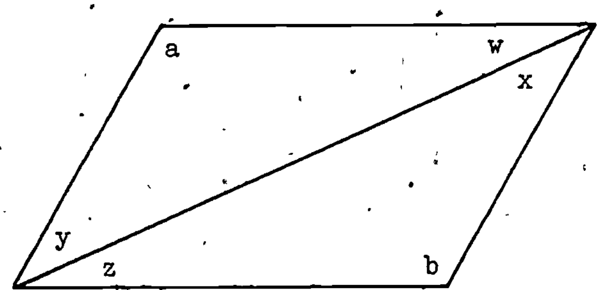
   (c) If the measure of $\overline{BC}$ is 5 , what is the measure of $\overline{DA}$?

   ____

   (d) If the measure of $\overline{AB}$ is 8 , what is the measure of $\overline{CD}$?

   ____

2. The figure to the right is a parallelogram. $\angle w$ and $\angle z$ are a pair of alternate interior angles and $\angle x$ and $\angle y$ are a pair of alternate interior angles.

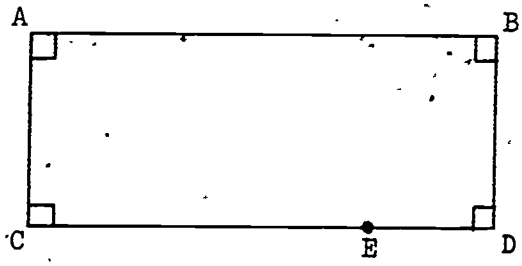


   (a) If $m \angle w = 30$, what is $m \angle z$? ________

   (b) If $m \angle y = 40$, what is $m \angle x$? ________

   (c) If $m \angle a + m \angle b = 220$, what must be the measure of each of these angles? ________

3. The figure below is a rectangle.



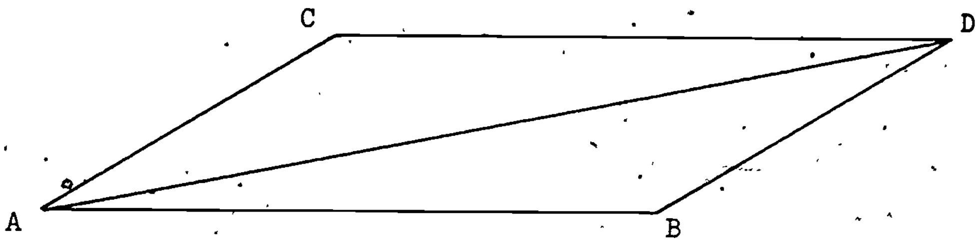
(a) Draw $\overline{BE}$.

(b) Suppose you were to cut along $\overline{BE}$ and slide the triangular piece EBD so that $\overline{BD}$ "butted up against AC ". What kind of figure would be formed? ________________

(c) Make a tracing of the rectangle above and do the experiment.

BRAINBOOSTER.

4. The figure below is a parallelogram with one of its diagonals drawn in.



Use the SSS property of congruent triangles and show (by writing a few sentences of explanation) why

$$\triangle ADC \cong \triangle ADB \text{ .}$$

____

____

____

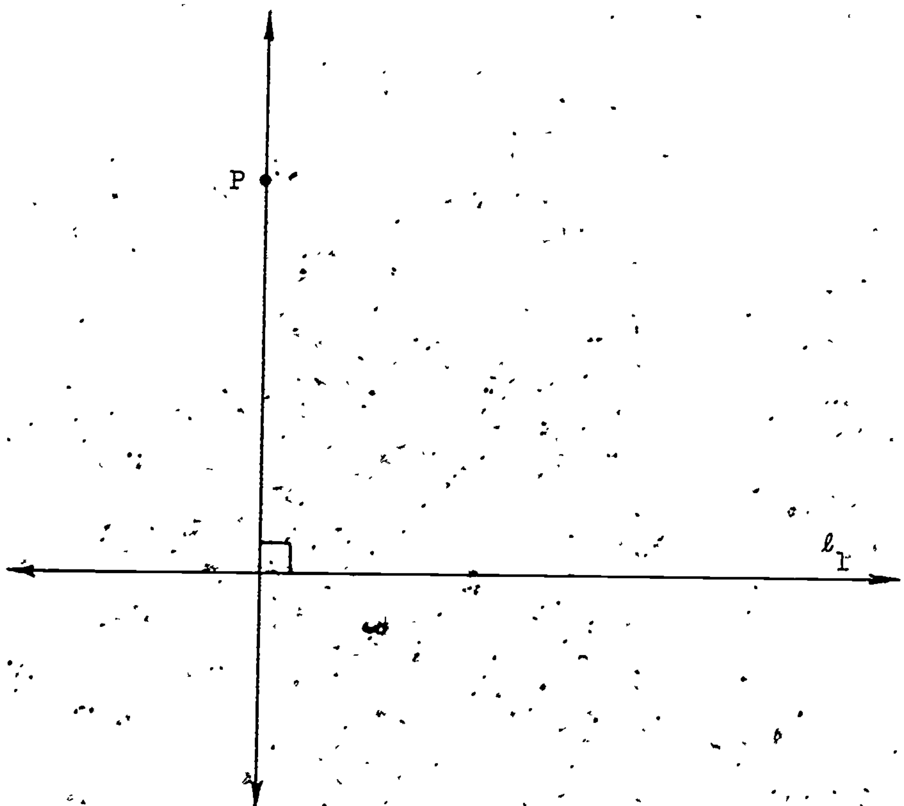
____

Pre-Test Exercises

Listed below are the types of problems and constructions you will have on the chapter test. If you don't know how to do them, read the section again. If you still don't understand, ask your teacher.

1. (Section 11-3.)

Construct a line through point P parallel to ℓ_1.



2. (Section 11-3.)

How many lines can pass through point P that are

(a) parallel to ℓ_1 ? ________

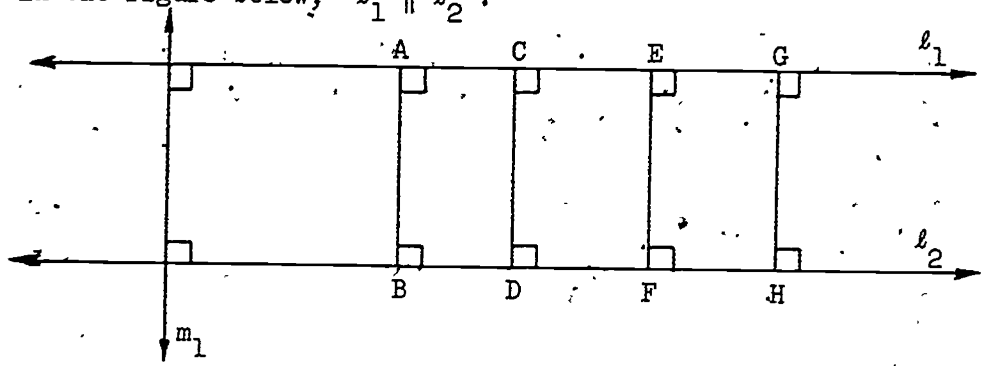
(b) perpendicular to ℓ_1 ? ________

3. (Section 11-3.)

The symbol "$\|$" means "_______________________________".

4. (Section 11-4.)

In the figure below, $\ell_1 \| \ell_2$.

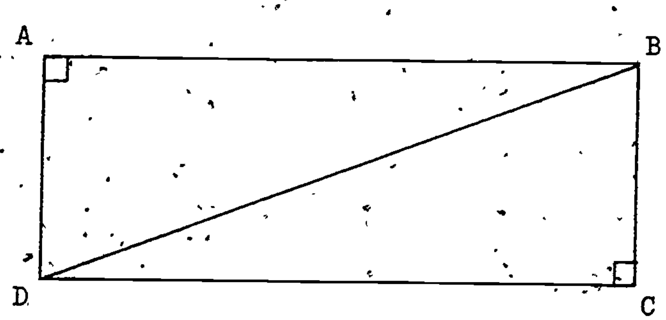


(a) Segments $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$ are all ________________ to each other.

(b) Segments $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$ are perpendicular to both __________ and __________.

5. (Section 11-4.)

The figure below is a rectangle with a diagonal drawn.



Fill in the blanks with the correct answer.

(a) $\overline{AB} \cong$ __________

(b) $\overline{AD} \cong$ __________

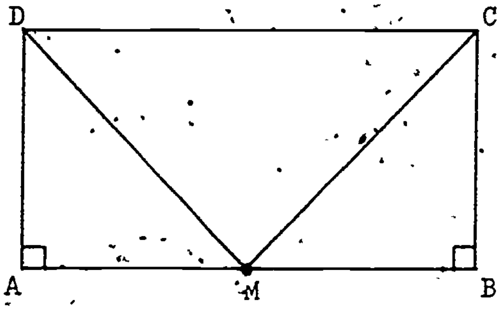
(c) $\overline{DB} \cong$ __________

(d) Then, by the SSS property of triangles,

$\triangle ABD \cong$ _______________

6. (Section 11-4.)

The figure below is a rectangle. Point M is the midpoint of $\overline{AB}$.



The following congruences are true.

(a) $\overline{MA} \cong \overline{MB}$

(b) $\angle MAD \cong \angle MBC$

(c) $\overline{AD} \cong \overline{BC}$

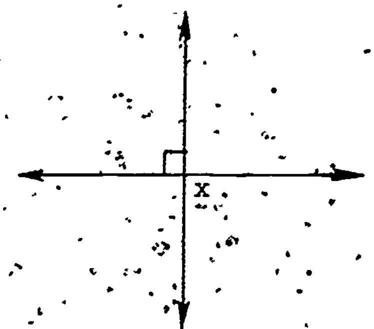
By the ____________ congruence property, $\triangle ADM \cong \triangle BCM$.

7. (Section 11-5.)

The measure of a straight angle is ________.

8. (Section 11-5.)

In the figure to the right, what is the measure of $\angle x$? ________

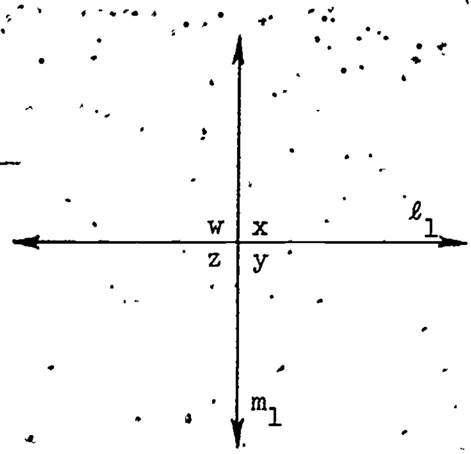


9. (Section 11-5.)

In the figure to the right, $\angle w$, $\angle x$, $\angle y$, and $\angle z$ are all congruent to each other. Then m_1 must be

____

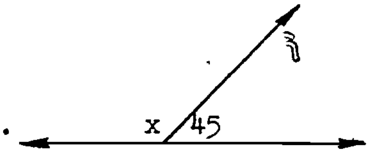
to ℓ_1.



10. (Section 11-5.)

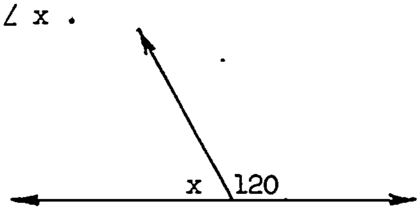
In each figure, find the measure of $\angle x$.

(a)



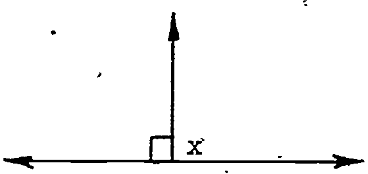
$m \angle x =$ ________

(c)



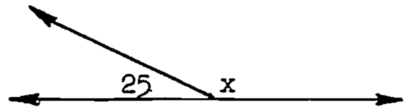
$m \angle x =$ ________

(b)



$m \angle x =$ ________

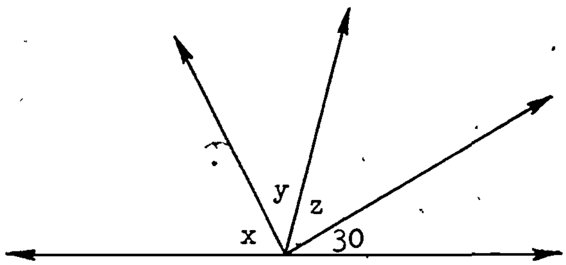
(d)



$m \angle x =$ ________

11. (Section 11-5.)

In the figure to the right,



(a) What is the sum

$m \angle x + m \angle y + m \angle z + 30$?

____

(b) What is the sum

$m \angle x + m \angle y + m \angle z$?

____

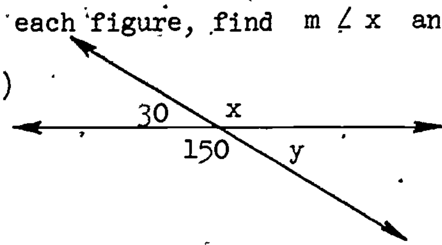
12. (Section 11-6.)

Any pair of <u>vertical</u> <u>angles</u> are ________________.

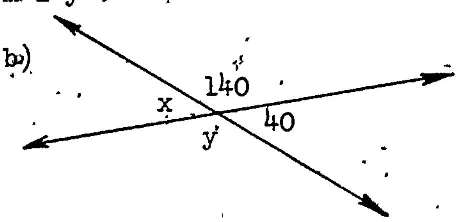
13. (Section 11-6.)

In each figure, find $m \angle x$ and $m \angle y$.

(a)



(b)



$m \angle x =$ ________

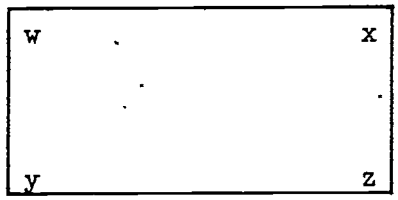
$m \angle y =$ ________

$m \angle x =$ ________

$m \angle y =$ ________

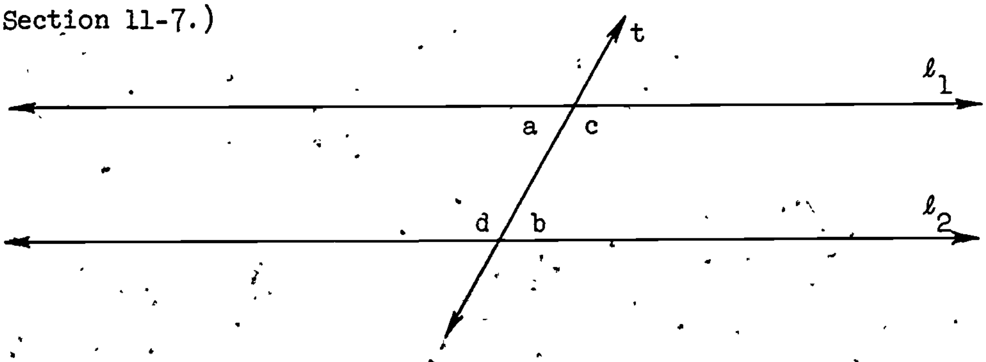
14. (Section 11-6.)

The figure below is a rectangle.



$m\angle w + m\angle x + m\angle y + m\angle z =$ ________.

15. (Section 11-7.)



In the figure above, $\ell_1 \parallel \ell_2$.

(a) Name a pair of alternate interior angles: ________________.

(b) Alternate interior angles have ________________ measure.

16. (Section 11-7.)

Use the drawing at the right, where $\ell_1 \parallel \ell_2$, to find the following measures.

(a) $m\angle a =$ ________

(b) $m\angle b =$ ________

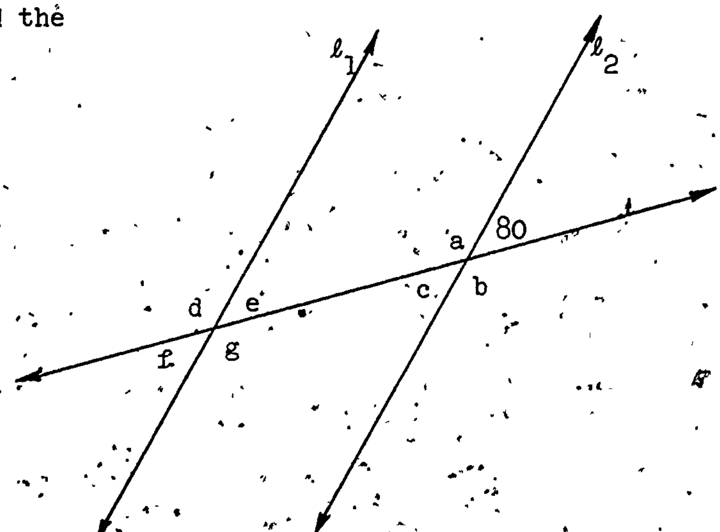
(c) $m\angle c =$ ________

(d) $m\angle d =$ ________

(e) $m\angle e =$ ________

(f) $m\angle f =$ ________

(g) $m\angle g =$ ________



17. (Section 11-8.)

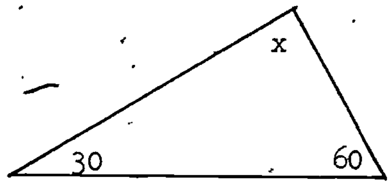
(a) The sum of the measures of the angles of a triangle is ________.

(b) This is the same as the measure of a ____________ angle.

18. (Section 11-8.)

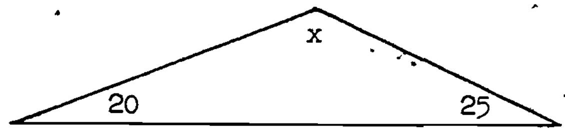
In each case find the measure of $\angle x$.

(a)



$m \angle x =$ ________

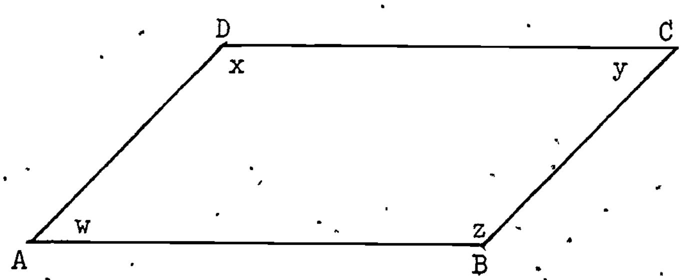
(b)



$m \angle x =$ ________

19. (Section 11-9.)

The figure below is a parallelogram.



(a) $m \angle w = m \angle$ ____

(b) $m \angle$ ____ $= m \angle x$.

(c) Opposite angles of a parallelogram are ____________ in measure.

(d) $\overline{AD} \cong$ ________

(e) ________ $\cong \overline{DC}$

(f) Opposite sides of a parallelogram are ____________ in measure.

(g) $\overline{AD} \parallel$ ________

(h) ________ $\parallel \overline{AB}$

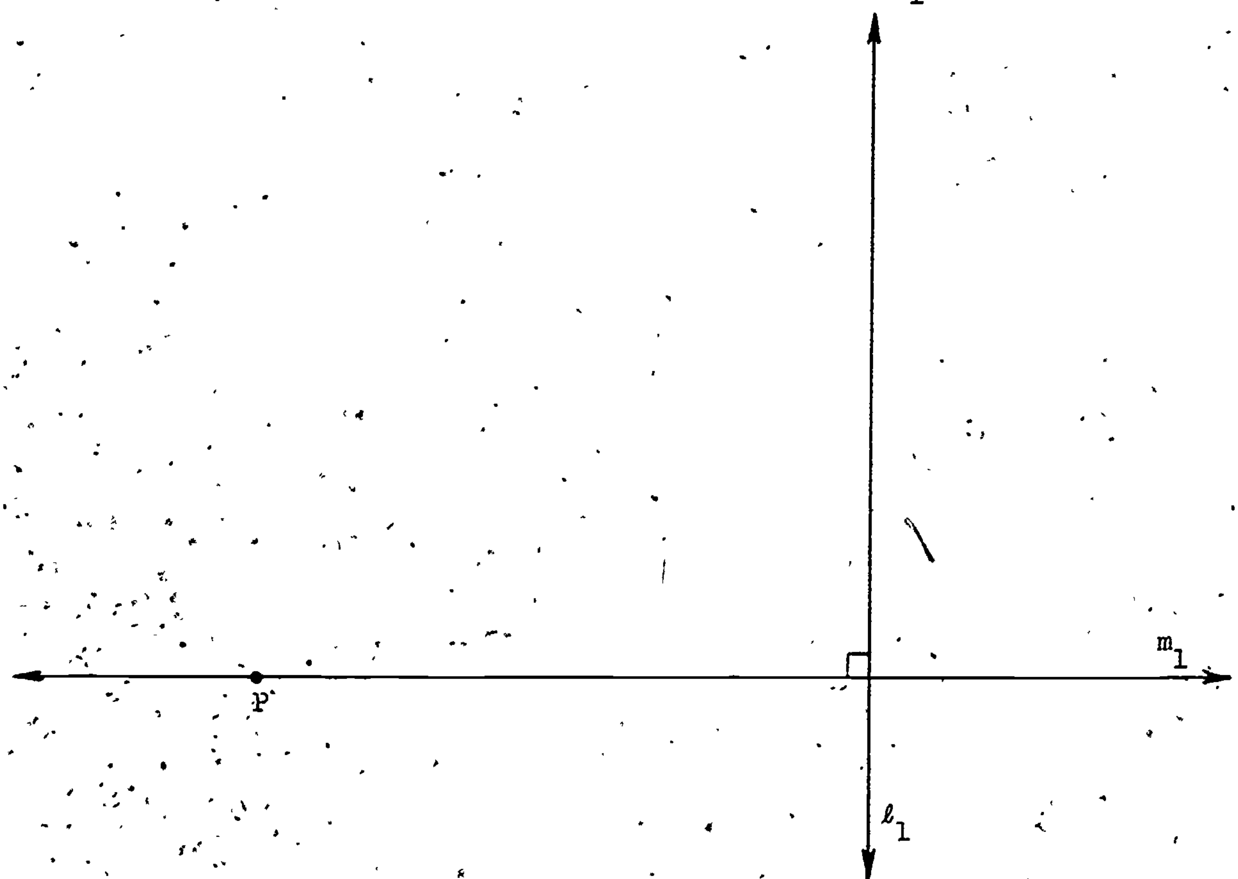
(i) Opposite sides of a parallelogram are ____________.

(j) $m \angle w + m \angle x + m \angle y + m \angle z =$ ________.

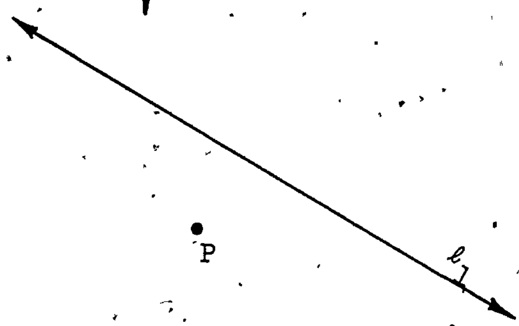
TEST

Chapter 11

1. Construct a line through point P parallel to ℓ_1.

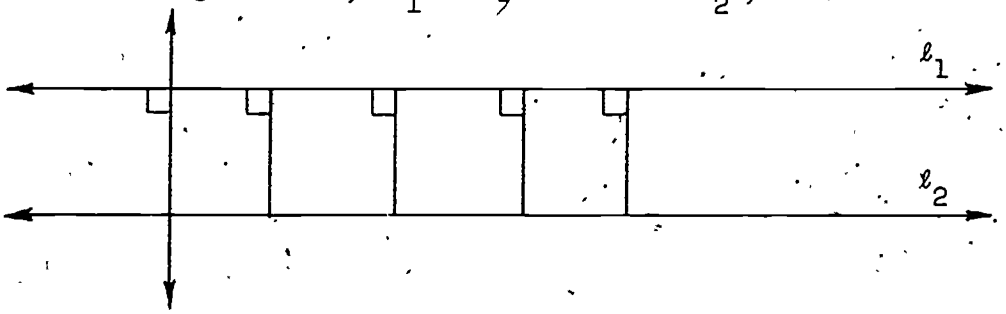


2. How many lines can pass through point P that are:
   (a) parallel to ℓ_1 ? ________
   (b) perpendicular to ℓ_1 ? ________



3. If we want to write in symbols that ℓ_1 __is parallel to__ ℓ_2 we would write: ℓ_1 ______ ℓ_2.

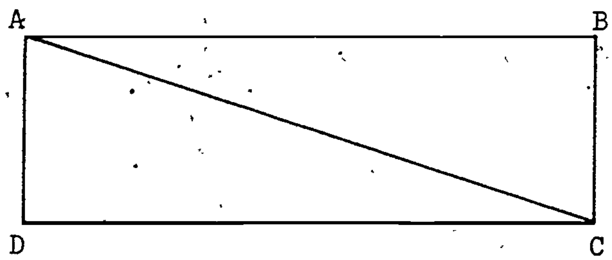
4. In the figure below, ℓ_1 is parallel to ℓ_2, so:



(a) all of the perpendicular line segments are ______________ to each other.

(b) ℓ_1 and ℓ_2 are everywhere the ____________ distance apart.

5. The figure below is a rectangle with a diagonal drawn.



(a) $\overline{AD} \cong$ ________

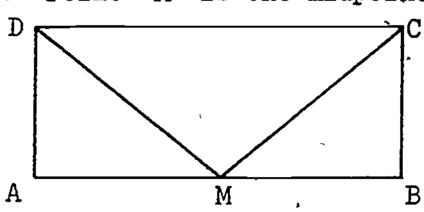
(b) $\overline{DC} \cong$ ________

(c) $\overline{AC} \cong$ ________

(d) Then, by the ________ property of triangles,

$\triangle$________ $\cong$ $\triangle$________

6. The figure below is a rectangle. Point M is the midpoint of $\overline{AB}$; thus $\overline{MA} \cong \overline{MB}$.



(a) $\overline{MA} \cong \overline{MB}$

(b) ______ $\cong \angle MBC$

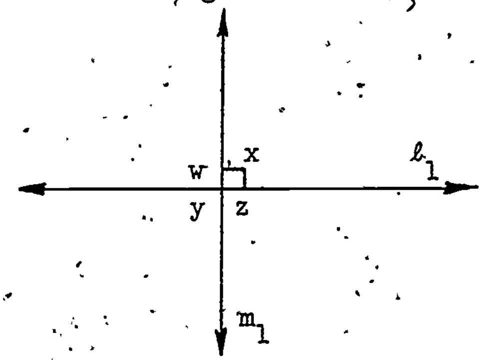
(c) $\overline{DA} \cong$ ______

(d) Then, by the ____________ congruence property of triangles,

$\triangle$________ $\cong$ $\triangle$________

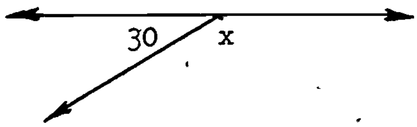
7. If an angle has a measure of 180 , then the angle must be a ______________ angle.

8. In the figure to the right, if ℓ_1 is perpendicular to m_1 then angles w , x , y , and z must all be ______________ to each other.



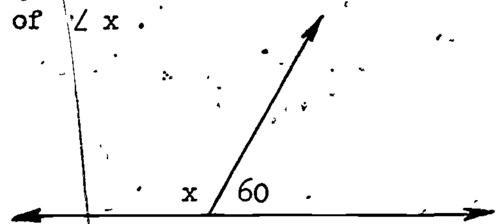
9. In each figure below, find the measure of $\angle x$.

(a)



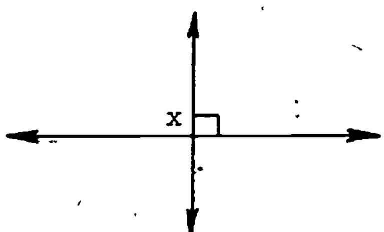
$m \angle x =$ ______________

(c)



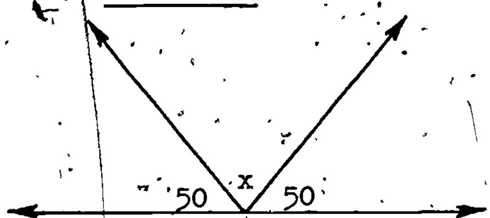
$m \angle x =$ ______________

(b)



$m \angle x =$ ______________

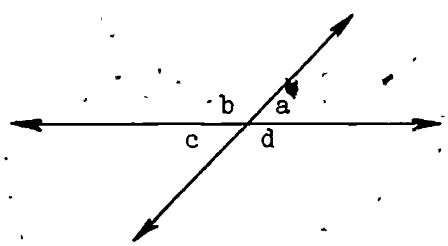
(d)



$m \angle x =$ ______________

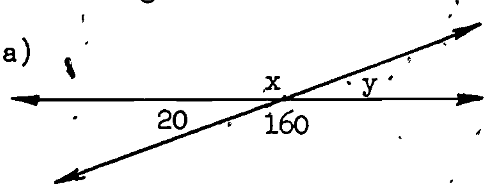
10. In the figure to the right name a pair of vertical angles.

$\angle$ ______ and $\angle$ ______



11. In each figure find $m \angle x$ and $m \angle y$.

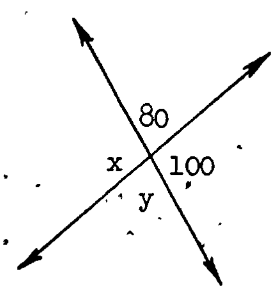
(a)



$m \angle x =$ ______________

$m \angle y =$ ______________

(b)



$m \angle x =$ ______________

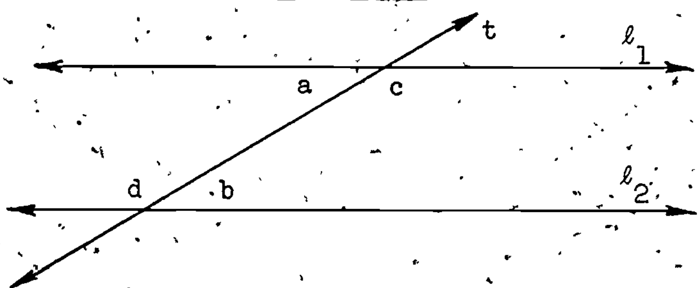
$m \angle y =$ ______________

12. The figure below is a rectangle.

The measure of each of the angles w , x , y , and z is ________.

13. In the figure below $\ell_1 \parallel \ell_2$. Name a pair of alternate interior angles.

________ and ________



14. Use the drawing at the right, where $\ell_1 \parallel \ell_2$, to find the following measures.

(a) $m\angle a$ = ________

(b) $m\angle b$ = ________

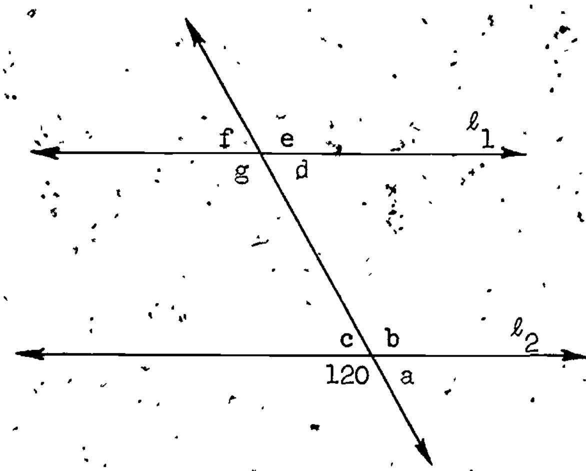
(c) $m\angle c$ = ________

(d) $m\angle d$ = ________

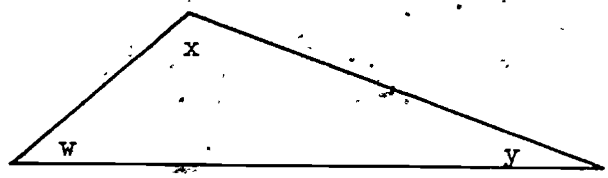
(e) $m\angle e$ = ________

(f) $m\angle f$ = ________

(g) $m\angle g$ = ________



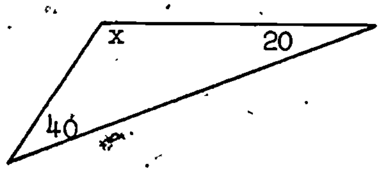
15. In the figure below,



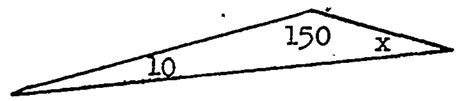
m ∠ w + m ∠ x + m ∠ y = ________.

16. In each case find the measure of ∠ x .

(a)

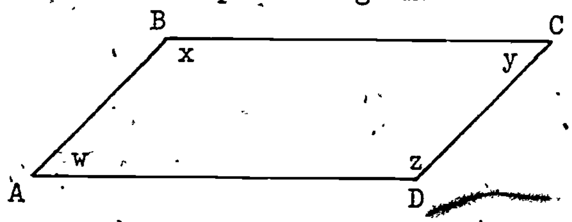


(b)



m ∠ x = ________ m ∠ x = ________

17. The figure below is a parallelogram.



(a) Name two angles that have equal measure.

∠ ________ and ∠ ________

(b) Name two line segments that have equal measure.

________ and ________

(c) What is the sum of the measures of the angles of a parallelogram? ________

Check Your Memory: Self-Test

1. (Section 7-8.)

Finish the following tables to show the outcomes from tossing three coins.

First Coin (rows) / Second Coin (columns):

	H	T
H		
T		

First and Second Coins (rows) / Second Coin (columns):

	H	T
HH	HHH	HHT
HT		

For three coins:

(a) P(2 heads) = ________

(b) P(at least 2 heads) = ________

(c) P(no heads) = ________

(d) P(at least 1 head) = ________

(e) Without making another table, find the probability of 4 heads if you toss 4 coins. ________

2. (Section 8-7.)

Match each equation on the left with an equivalent equation on the right by writing the correct equation on the line.

(a) $5x + 7 = {}^{-}3$ ________

(b) $5x + 10 = 5$ ________

(c) $\frac{1}{2}x + 3 = 5$ ________

(d) $\frac{1}{2}x + 4 = 9$ ________

(e) $\frac{1}{2}x = 3$ ________

$5x = {}^{-}5$

$x = 6$

$\frac{1}{2}x = 2$

$5x = {}^{-}10$

$\frac{1}{2}x = 5$

3. (Section 10-2.)

Divide.

(a) $7\overline{)4882}$ (c) $5\overline{)499}$

(b) $25\overline{)3450}$ (d) $8\overline{)9608}$

4. (Section 10-9.)

Give the decimal name for each number.

(a) $\frac{3}{4}$ = ________   (d) $\frac{5}{2}$ = ________

(b) $\frac{1}{2}$ = ________   (e) $\frac{9}{10}$ = ________

(c) $\frac{1}{8}$ = ________   (f) $\frac{1}{5}$ = ________

5. (Sections 10-11, 10-12, and 10-13.)

Find the answers. Watch the signs!

(a) $.395 + 2.14$ = ________

(b) $3.15 - 2.786$ = ________

(c) $.07 \times .5$ = ________

(d) $\frac{.6}{.03}$ = ________

Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1.

First Coin \ Second Coin	H	T
H	HH	HT
T	TH	TT

First and Second Coins \ Third Coin	H	T
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

(a) P(2 heads) = $\frac{3}{8}$

(b) P(at least 2 heads) = $\frac{4}{8}$ or $\frac{1}{2}$

(c) P(no heads) = $\frac{1}{8}$

(d) P(at least 1 head) = $\frac{7}{8}$

(e) If you toss 4 coins, P(4 heads) = $\frac{1}{16}$

$\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)$

2. (a) $5x + 7 = {}^-3$

$5x = {}^-10$

(b) $5x + 10 = 5$

$5x = {}^-5$

(c) $\frac{1}{2}x + 3 = 5$

$\frac{1}{2}x = 2$

(d) $\frac{1}{2}x + 4 = 9$

$\frac{1}{2}x = 5$

(e) $\frac{1}{2}x = 3$

$x = 6$

3. (a) $7\overline{)4\,8\,8\,2}$ with quotient $697\frac{3}{7}$

(c) $5\overline{)4\,9\,9}$ with quotient $99\frac{4}{5}$

(b) $25\overline{)3450}$ with quotient 138

$$\begin{array}{r} 3450 \\ \underline{2500} \\ 950 \\ \underline{750} \\ 200 \\ \underline{200} \end{array}$$

(d) $8\overline{)9608}$ with quotient 1201

4. (a) $\frac{3}{4} = .75$

(b) $\frac{1}{2} = .5$

(c) $\frac{1}{8} = .125$

(d) $\frac{5}{2} = 2.5$

(e) $\frac{9}{10} = .9$

(f) $\frac{1}{5} = .2$

5. (a) $.395 + 2.14 = 2.535$

(b) $3.15 - 2.786 = .364$

(c) $.07 \times .5 = .035$

(d) $\frac{.6}{.03} = 20$